1/15/20 Lecture outline

⋆ **Reading: Taylor Chapter 9.**

• Next topic: mechanics in non-inertial frames. Recall that Newton's first and second law apply in inertial reference frames. Any reference frame moving at a constant velocity with respect to an inertial frame is also inertial. When we discuss relativity, we will emphasize that physics in all inertial frames is physically equivalent. In this chapter, we are interested, however, in non-inertial frames. Given an inertial frame, a non-inertial frame has a relative acceleration, a relative rotation, or both. Discuss these cases in turn.

Let $\vec{r}(t)$ be the position of an object of mass m in an inertial reference frame. Let $\vec{R}$ be a vector from the origin if the inertial frame to the origin of another frame of reference, which we will call the prime frame (not Amazon). The position in the prime frame is then $\vec{r}' = \vec{r} - \vec{R}$. The velocity in the prime frame is $\vec{v}' = \vec{v} - \vec{V}$. The acceleration in the prime frame is $\vec{a}' = \vec{a} - \vec{A}$. We will modify these transformations when we discuss relativity; they only apply as given for velocities $v, v', V \ll c$. Note that $m\vec{a}' = \vec{F} - m\vec{A} \equiv \vec{F} + \vec{F}_{intertial}$.

Example: a pendulum in an accelerating car has $m\vec{a}' = \vec{T} + m(\vec{g} - \vec{A})$ where $\vec{T}$ is the tension in the string. The equilibrium angle is that of a triangle with adjacent side g and opposite side A, i.e. $\phi_{eq} = \tan^{-1}(A/g)$. For fun, consider a balloon in an accelerating car.

In terms of the Lagrangian, we have $L = \frac{1}{2}m\dot{\vec{r}}^2 - U(\vec{r}, t) = \frac{1}{2}m(\dot{\vec{r}'} + \dot{\vec{R}})^2 - U(\vec{r}' + \vec{R}, t) \cong \frac{1}{2}m\dot{\vec{r}'}^2 - m\vec{r}' \cdot \vec{A} - U(\vec{r}' + \vec{R}, t)$ where in the last step we dropped total derivative terms, as is allowed. We see that there is an additional force term $\vec{F}_{fict} = -m\vec{A}$.

• Example of tides: the force from the moon on some water of mass m is $\vec{F}_{m\to moon} = -GM_{moon}m\vec{d}/d^3$, where $\vec{d}$ is a vector from the center of the moon to the mass m. The mass is attached to the reference frame of the earth, which is accelerated by the force of the moon, $\vec{F}_{earth\to moon} = -GM_{moon}M_{earth}\vec{d}_0/d_0^3 = M_{earth}\vec{A}$ where $\vec{d}_0$ is a vector from the center of the moon to the center of the earth, and $\vec{A}$ is the acceleration of the center of the earth to the moon. So the mass m has $m\vec{a}' = \vec{F}_{m\to moon} - \frac{m}{M_{earth}}\vec{F}_{earth\to moon}$. This makes the water bulge out on both the near and far sides from the moon, and flatten on top. Write $F_{tid} = \vec{F}_{m\to moon} - \frac{m}{M_{earth}}\vec{F}_{earth\to moon} = -\nabla U_{tid}$, with $U_{tid} = (-GmM_{moon}d^{-1} + GmM_{moon}d_0^{-2}x)$ and the liquid surface is an equipotential of U_{tid}. Use to estimate the height, $h \approx 3M_mR_e^4/2M_ed_0^3 \approx 0.5m$ from the moon. For the case of the sun replace $M_m \to M_s$ and $d_0 \to$ the distance to the sun, get $h \approx 0.25m$ from the sun. It is thus important to include the effect of the sun, which explains the large tide swings when the moon is collinear with the earth and sun. Also, we should include rotation effects.