3/2/20 Lecture outline

⋆ **Reading: Taylor chapter 15.**

• Last time: electromagnetism is nicely relativistic: $J^\mu = (c\rho, \vec{J})$ transforms as a 4-vector. The electric and magnetic fields combine into $F^{\mu\nu}$ as $F^{i0} = E^i$ and $F^{ij} = -\epsilon^{ijk}B_k$, and then $E_x = E'_x$, $B_x = B'_x$, and

$$\begin{pmatrix} E_y \\ B_z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E'_y \\ B'_z \end{pmatrix}, \quad \begin{pmatrix} E_z \\ B_y \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E'_z \\ B'_y \end{pmatrix}.$$

The Lorentz force law is indeed relativistic: $\frac{dp^\mu}{d\tau} = f^\mu = \frac{q}{c}F^{\mu\nu}u_\nu$.

• Write Maxwell's equations as $\partial_\mu F^{\mu\nu} = \frac{4\pi}{c}J^\mu$ and $\partial_\mu \tilde{F}^{\mu\nu} = 0$, where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. The second equations (absence of magnetic charges) can be solved via $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Gauge transformation symmetry (actually, it's a redundancy): can replace $A^\mu(x) \to A^\mu(x) + \partial^\mu f(x)$ for any $f(x)$ without changing any physical quantity. Very important in the quantum description of E and M.

• The force law comes from including a term $S \supset -\frac{q}{c}\int A_\mu dx^\mu$ which gives $L \supset -q\phi + \frac{q}{c}\vec{v}\cdot\vec{A}$. Gives e.g. $\vec{p} = \frac{\partial L}{\partial \vec{v}} = \gamma m\vec{v} + \frac{q}{c}\vec{A}$ and then $H = \vec{p}\cdot\vec{v} - L = \sqrt{(c\vec{p} - q\vec{A})^2 + (mc^2)^2} + q\phi$.

The combinations $\vec{E}^2 - \vec{B}^2 \sim F_{\mu\nu}F^{\mu\nu}$ and $\vec{E}\cdot\vec{B} \sim \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ are Lorentz invariant. Can get Maxwell's equations from least action in the field A_μ with $S \supset \int d^4x\mathcal{L}$ with $\mathcal{L} = -\frac{1}{16\pi c}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}A_\mu J^\mu$; note that S is Lorentz invariant. Least action for fields will be discussed soon.