2/26/20 Lecture outline

⋆ **Reading: Taylor chapter 15.**

• Last time: Lorentz transformations are a symmetry of Nature, and this is ensured by having the action be Lorentz invariant. Let's start with a free, massive particle; we want to generalize $S \approx \int dt(\frac{1}{2}m\vec{v}^2)$. To get a Lorentz invariant, we can take $S = -mc^2 \int_{worldline} d\tau = -mc^2 \int dt\sqrt{1-\vec{v}^2/c^2}$. Gives $\vec{p} = \frac{\partial L}{\partial \vec{v}} = \gamma m\vec{v}$ and properly reduces to $L \approx \frac{1}{2}m\vec{v}^2$ if $v \ll c$.

The Hamiltonian is $H = \vec{p}\cdot\vec{v} - L = \gamma m\vec{v}^2 + \gamma^{-1}mc^2 = \gamma mc^2 = \sqrt{(cp)^2 + mc^2}$. Verify that Hamilton's equations are satisfied, e.g. $\vec{v} = \partial H/\partial\vec{p} = c^2\vec{p}/E$.

• A traveling plane wave can be written as $\psi(t,\vec{x}) = e^{i(\vec{k}\cdot\vec{x}-\omega t)} = e^{-ik_\mu x^\mu}$ (or its real or imaginary part). The phase factor is properly Lorentz invariant since $k^\mu = (\omega/c, \vec{k})$ transforms as a 4-vector. The Lorentz transformation of k^μ gives the relativistic Doppler formula: in the prime frame, get $\omega' = \gamma\omega - \beta\gamma k_x$ and $k_{x'} = -\beta\gamma\omega + \gamma k_x$. For light we have $\omega = ck$ and $\omega' = ck'$. For example, for a light ray traveling along the x axis, taking $\omega' = \omega_0$, then $\omega = \gamma(1+\beta)\omega_0 = \omega_0/\gamma(1-\beta) = \omega_0\sqrt{\frac{1+\beta}{1-\beta}}$. Here β is the relative speed of the source towards the receiver. Contrast this with the non-relativistic Doppler effect for waves traveling in a medium. Suppose that the source is moving with velocity $v_s\hat{x}$ relative to the rest frame of the medium, and is at negative x relative to the observer, and that the observer is moving with velocity $v_o\hat{x}$ relative to the medium (source is moving towards the observer, and the observer is moving away from the source). The non-relativistic Doppler formula is $\omega_{obs}^{NRD} = (\frac{1-v_r/c_m}{1-v_s/c_m})\omega_{source}$, where $c_m = \omega/k$ is the speed of wave propagation in the medium. It does not depend only on v_{rel} because the air determines a rest frame. For $c_m \gg v_{r,s}$ we get $\omega_{obs}^{NRD} \approx (1 + v_{rel}/c_m)\omega_{source}$ with $v_{rel} = v_s - v_r$. For $\beta \ll 1$, the relativistic Doppler formula is similar: $\omega \approx (1+\beta)\omega_0$.

• Electromagnetism is nicely relativistic. The source terms in Maxwell's equations organize into a 4-vector: $J^\mu = (c\rho, \vec{J})$. Charge conservation of $Q = \int d^3x\rho$ and current conservation. It can be written as $\partial_\mu J^\mu = 0$, so it is Lorentz invariant. Next time: $Q = Q'$ is Lorentz invariant.