2/14/20 Lecture outline

⋆ **Reading: Taylor sections 15.4 to 15.14.**

• Last time: Lorentz transformations preserve $ds^2 = (cdt)^2 - d\vec{x}^2$. Aside: these transformations are called $SO(1,3)$. They are given by rotations of vectors (e.g. the three Euler angles), and boosts along the three axes. The form of a boost along the $\hat{x}$ axis is $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$ with $\beta = v/c$ and $y' = y$, $z' = z$.

Proper time is the time that one records on their own personal clock, in their rest frame (the frame where $d\vec{x}' = 0$): $ds^2 = (cdt)^2 - d\vec{x}^2 \equiv (cd\tau)^2$. So $d\tau \equiv dt_{proper} = dt\sqrt{1 - v^2/c^2} = dt/\gamma$. Show this for the example of a bouncing light ray as seen in two frames. Proper-time $d\tau = dt\sqrt{1 - v^2/c^2}$ works even in accelerating frames. The total proper time elapsed is $\Delta\tau = \int_{worldline} d\tau$ where the integral is over their worldline in spacetime. Consider two twins' worldlines: the homebody one just sits there, and the mover one goes at velocity $\vec{v} = v\hat{x}$ and then turns around with velocity $-v\hat{x}$ and returns. Get $\Delta\tau_{mover} = \sqrt{1 - v^2/c^2}\Delta\tau_{homebody} < \Delta\tau_{homebody}$. Note that the proper time is longest for the twin with the straight worldline: a straight line in spacetime has the longest, not the shortest proper time.

• Likewise, proper length = the length in an object's rest frame. Consider a stick that is at rest in the $'$ frame, of length $\Delta x'$. In the lab we measure the length by recording the position of both ends simultaneously, at $\Delta t = 0$, Then $\Delta s^2 = -\Delta x^2 = (c\Delta t')^2 - (\Delta x')^2 = -(\Delta x')^2/\gamma^2$ where the time separation is because $dt = 0$ requires $cdt' = -\beta dx'$. Note that the measurement of the two ends in the lab frame is simultaneous, $dt = 0$, but not in the prime frame; the notion of simultaneous differs in the frames. So $\Delta x = \Delta x'/\gamma$: length contraction. The proper length element is $dx_{proper} = \gamma dx$.

• Example of a pole vaulter and a barn, to illustrate that both see length contraction. The runner in the pole frame sees the barn contracted (so both doors could not possibly be closed simultaneously with the pole inside the barn), and the person in the barn frame sees the pole contracted (so both doors can be simultaneously shut with the pole inside – briefly before the crash). There is no contradiction because they differ in what is measured, because the notion of simultaneity differs in the frames.

• Example: consider an unstable muon particle which decays in $10^{-6}s$ and which is moving with velocity v relative to the lab frame. Without relativity, one would think that the particle can travel $v \cdot 10^{-6}$ meters before it decays, e.g. if $v \approx c$ then this is $\sim 300m$. According to relativity, and agreeing with observation, their proper lifetime of

$\Delta t' = \Delta\tau = 10^{-6}s$ becomes, in the lab frame, time dilated to $\Delta t = \gamma\Delta\tau$ so they can travel distance $\Delta x = v\gamma\Delta\tau$ before decaying. From the muon's perspective, their lifetime is $\Delta\tau$, and they travel this length because it is length contracted to $\Delta x' = \Delta x/\gamma = v\Delta\tau$, so it is consistent from either perspective.