2/10/20 Lecture outline

⋆ **Reading: Taylor sections 15.1, 15.2, 15.3, 15.10.**

• Symmetries (or lack thereof) give a wonderful bounty of physical insights. Recall (from Noether's theorem) the connection between symmetries and conservation laws. Since there is no preferred origin of space, physics is invariant under $\vec{x} \to \vec{x} + \vec{x}_0$ where $\vec{x}_0$ is any constant, and this implies conservation of momentum $\vec{P}_{tot}$ (as also seen from Newton's 3rd law for an isolated system). Likewise, physics is invariant under $t \to t + t_0$ and that is equivalent to conservation of E_{tot}. Invariance under rotation (e.g. by the Euler angles $\vec{\phi} \to \vec{\phi} + \vec{\phi}_0$ is a symmetry of Nature and leads to conservation of angular momentum $\vec{L}$. There is another symmetry of Nature that we'll be discussing in the coming lectures: symmetry under boosts. Physics is equivalent in frames that differ only by a constant relative velocity. This symmetry does not really lead to new, independent conserved quantities, but it nevertheless has many important (and strange, for v outside of our everyday intuition range!) consequences. Another symmetry is that physics in an accelerated frames is equivalent to gravity; this is Einstein's equivalence principle, which we will briefly mention later.

• We emphasized that Newton's laws are simplest in inertial frames, and any frame moving with a constant velocity relative to an inertial frame is also inertial. Let (t, x, y, z) be the coordinates of a spacetime event as seen in the lab frame, and (t', x', y', z') be the coordinates of the event as seen in some other frame that is moving at some relative velocity $\vec{v}$. We can choose our x axis such that $\vec{v} = v\hat{x}$, and take the two frames to coincide at the origin of both. So $x' = 0$ has $x = vt$. The Galilean transformation between the frames is $x' = x - vt$, $y' = y$, $z' = z$, $t' = t$. This transformation is a symmetry of Newton's laws: the observers in the two frames will measure identical physics from $\vec{F} = m\frac{d^2\vec{x}}{dt^2}$ since the accelerations are the same in the two frames. Velocities in the two frames are related by $\frac{dx'}{dt} = \frac{dx}{dt} - v$.

• The principle of relativity: suppose that you and your friend are each in an inertial frame, moving with constant relative velocity v, then no physics experiment can determine which frame is moving. Of course, we saw that is not the case for accelerating frames, since they lead to additional fictitious forces. And it is not the case if there is some medium, like the air or the ocean, which has a preferred rest frame. But if everything is moving, or the observer is isolated from the outside, then no physics experiment would be able to distinguish the frames.

• Einstein noticed that Maxwell's equations do not have Galilean transformations as a symmetry; instead, they have a different symmetry. The lack of Galilean symmetry suggests a violation of the relativity principle in E and M, and that was expected pre-Einstein times because E and M leads to light waves, and physicists thought that waves require a medium, like water waves in the ocean, and then that medium (called the aether) would have a preferred rest frame.

Einstein's equations in vacuum imply, for example, that $\vec{E}$ satisfies the wave equation $(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\psi = 0$, where ψ is any component of the $\vec{E}$ or $\vec{B}$ field. This leads to traveling wave solutions, like $\psi = \psi_0 \cos(\omega t - \vec{k} \cdot \vec{x})$ where $\omega = ck$, the wave moves at speed $c = 1/\sqrt{\mu_0 \epsilon_0} \approx 3 \times 10^8 m/s$. This cannot be satisfied in both of the frames related by Galilean relativity because of the above classical velocity addition formula. Indeed, the Galilean relations and the chain rule lead to $\frac{\partial}{\partial x} = \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t} = -v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}$ and then the wave function becomes complicated in the prime frame, with solutions moving at speed $c_{R,L} = c \mp v$.

Initially, physicists thought that the interpretation was that the wave moves at speed c only in some preferred frame – the rest frame of the aether, and then Galilean relativity would be wrong for an interesting reason: one could measure motion relative to the aether. Michelson-Morley tried to measure this with an interferometer, and they got a null result.

Einstein's brilliant realization was that there is no aether, and that relativity is right – every frame moving at constant relative velocity is physically equivalent, and that Maxwell's equations are right and true in every frame. But the different frames are not related by the above Galilean relation – it is just an approximation (in v/c) to the correct transformation between the frames. The correct transformation is called a Lorentz transformation. Happily, Maxwell's equations already had the symmetry under Lorentz transformations built into them.

• Consider two infinitesimally separated spacetime events. Let $(dt, d\vec{x})$ be their separation in time and space. Consider $ds^2 = (cdt)^2 - d\vec{x} \cdot d\vec{x}$. If $ds^2 > 0$, the two events are said to be time-like separated. If $ds^2 < 0$, the two events are said to be space-like separated. If $ds^2 = 0$, the two events are said to be null-separated. According to Einstein's postulates, light rays move along $ds^2 = 0$ as seen in any frame: so if $0 = (cdt)^2 - d\vec{x} \cdot d\vec{x}$, then $0 = (cdt')^2 - d\vec{x}' \cdot d\vec{x}'$. It then follows that $(cdt)^2 - d\vec{x} \cdot d\vec{x} = (cdt') - d\vec{x} \cdot d\vec{x}$ always is the only consistent possibility. We could try $ds^2 = f(v) ds'^2$, but then $v \to -v$ should sent $f \to 1/f$ and the only possibility is $f = 1$.

ds^2 is called the invariant interval between spacetime events and we can find the form of the Lorentz transformations from the requirement that it is invariant. For motion along the x axis, can see that $dy' = dy$ and $dz' = dz$, so the transformation has $(cdt)^2 - dx^2 = (cdt')^2 - dx'^2$. If there were a relative + sign, it would be a rotation matrix and the dot product would be preserved using $\sin^2\theta + \cos^2\theta = 1$. The relative minus sign leads to a close cousin....(to be continued next time)