110b HW Due 2/28/20

1. Taylor 15.60 (only turn it in if you didn't in the last HW set).

2. In lecture, we discussed the Lorentz transformation Λ of a four vector a^μ under boosts with velocity v along the x-axis.

   (a) Verify that a_μ transforms by the inverse Λ^{-1}, which is related to Λ by $v \to -v$.

   (b) Using the chain rule, show that $\frac{\partial}{\partial x^\mu}$ transforms the same way as a_μ, with a lower index. So $\frac{\partial}{\partial x^\mu} \equiv \partial_\mu$.

   (c) Verify that the Lorentz boost along the x-axis satisfies $\Lambda^T \eta \Lambda = \eta$, where η is the flat metric of spacetime.

3. Consider the spacetime path $x \equiv x^1 = x_0(\cosh\lambda - 1)$, $ct = x_0 \sinh\lambda$, where λ is a coordinate along the spacetime worldline of the object.

   (a) Compute the proper time $d\tau$ for this path, and show that it is proportional to $d\lambda$, therefore λ is proportional to τ. Find the proportionality constant.

   (b) Compute the 4-velocity $u^\mu = \frac{dx^\mu}{d\tau}$ and $v \equiv \frac{dx}{dt}$ for this path, as a function of the proper time τ.

   (c) Compute the 4-acceleration $a^\mu = \frac{d^2x^\mu}{d\tau^2}$.

4. A rocket passes Earth, with velocity $\vec{v}_{rel} = 4c/5\widehat{x}$ relative to the Earth reference frame. A passenger on the rocket throws a ball with velocity $\vec{v}'_{ball} = (c/2)\,(\cos\theta'\widehat{x} + \sin\theta'\widehat{y}')$. Write the velocity $\vec{v}_{ball}$ as seen by an observer on Earth. Check that your answers are sensible for the special cases of $\theta' = 0$ and $\theta' = \pi/2$ and $\theta' = \pi$.

5. A particle of rest mass energy $m_1 = 3GeV$ and total energy $E_1 = 5GeV$ is traveling along the $+\widehat{x}$ axis. It collides head one with a particle of rest mass energy $m_2 = 4GeV$ and total energy $E_2 = 5GeV$, that was traveling along the $-\widehat{x}$ axis. The two particles fuse into a single particle. Write your answers in $c = 1$ units, with energy in GeV.

   (a) What is the energy E_3 and momentum $\vec{p}_3$ of that final state particle?

   (b) What is the mass m_3 of that final state particle?

   (c) What is the velocity $\vec{v}_3$ of the final state particle?