Physics 212b, Ken Intriligator lecture 7, Jan 31, 2018

• Continue with parity. As mentioned last time, P is represented by a unitary operator and if $[H, P] = 0$ then the theory has a parity symmetry, preserved by time evolution, and this leads to selection rules, $\langle\chi|\mathcal{O}|\psi\rangle = 0$ unless $(-1)^{P_\chi+P_\mathcal{O}+P_\psi} = 1$.

Note that if the theory respects parity and $|E_n\rangle$ is non-degenerate, then it is a parity eigenstate and the selection rule implies that $\langle E_n|\vec{v}|E_n\rangle = 0$ for any vector, e.g. zero dipole moment expectation value.

The groundstate is in general parity even. E.g. double SHO, (mention $e^{-S_{Euc}/\hbar}$ tunneling and instantons). find $E_A > E_S$. The non-stationary states $|R\rangle$ or $|L\rangle$, given by $(|S\rangle \pm |A\rangle)/\sqrt{2}$ are non-stationary, oscillating with frequency $\omega = (E_A - E_S)/\hbar$. Ammonia molecule NH_3 behaves like this, oscillating between N above vs below the triangle of the H_3. R vs L type molecules, with oscillation times like 10^6 years, organic material often one or the other, vs synthetic producing equal amounts.

In 3d, parity takes $\cos\theta \to -\cos\theta$ and $\phi \to \phi + \pi$, so $P|\alpha, \ell, m\rangle = (-1)^\ell|\alpha, \ell, m\rangle$. This is clear also from the $\ell = 1$ case, any getting general ℓ from tensor products. E.g. for the 3d SHO: the state with quantum numbers $|n_x, n_y, n_z\rangle$ has parity $(-1)^{n_x+n_y+n_z}$. This agrees with the fact that the parity is $(-1)^\ell$, since $\ell = n \bmod 2$.

• Time reversal, $t \to -t$, is a symmetry of usual classical physics. Time reversal in QM is an anti-unitary operator. E.g. if $\psi(\vec{x}, t)$ is a solution of the SE, then so is $\psi^*(\vec{x}, -t)$. The complex conjugation is the anti-unitary part. Writing the time reversal operator as T, it must be anti-unitary because we want $T^{-1}U(t)T = U(-t)$, but $T^{-1}HT = H$ if the theory is time-reversal invariant, so $T^{-1}HT \neq -H$. Time reversal acts as e.g. $T^{-1}\vec{x}T = \vec{x}$ and $T^{-1}\vec{p}T = -\vec{p}$, which again forces anti-unitarity, to preserve the commutation relations.

An anti-unitary linear operator acts as $T^{-1}(a\hat{A} + b\hat{B})T = a^*T^{-1}\hat{A}T + b^*T^{-1}\hat{B}T$. In terms of states $T(a|\psi\rangle + b|\chi\rangle) = a^*T|\psi\rangle + b^*T|\chi\rangle$. Can write $T = UK$ where K is complex conjugation. In position space this takes $\vec{x} \to \vec{x}$ and $\vec{p} \to -\vec{p}$ simply because $\vec{x}$ is real and $\vec{p} = -i\hbar\nabla$ is imaginary, so we can take $T = UK$ with $U = 1$ in position space.

$T|\vec{p}\rangle = |-\vec{p}\rangle$ and $T|\vec{x}\rangle = |\vec{x}\rangle$. If $|\psi\rangle = \int d^3\vec{x}|\vec{x}\rangle\langle\vec{x}|\psi\rangle$, then $T|\psi\rangle = \int d^3\vec{x}|\vec{x}\rangle\langle\vec{x}|\psi\rangle^*$.

$T\vec{J} = -\vec{J}T$, since the rotation operator and time reversal commute (and time reversal flips the i in the rotation operator). $T|\ell, m\rangle = (-1)^m|\ell, -m\rangle$, which is seen in the $Y_{\ell m}$ since T complex conjugates it. More generally, $T^2 = 1$ for integer j.