Physics 212b, Ken Intriligator lecture 6, Jan 29, 2018

• Parity P acts as $P = P^\dagger = P^{-1}$ with $P\vec{v}P = -\vec{v}$ and $P\vec{a}P = \vec{a}$ for vectors (e.g. $\vec{x}$ and $\vec{p}$) and axial pseudo-vectors (e.g. $\vec{L}$ and $\vec{B}$) respectively. Note that it is not a rotation. Parity preserves all the basic commutation relations, but it is not necessarily a symmetry of the Hamiltonian. The names scalar and pseudo scalar are often used to denote parity even vs odd: $P\mathcal{O}P = \mathcal{O}$ for "scalar" operators (like $\vec{x}^2$ or $\vec{L}^2$, and $P\mathcal{O}'P = -\mathcal{O}'$ for pseudo scalar operators (like $\vec{S}\cdot\vec{x}$).

• Nature respects parity aside from some tiny effects at the fundamental level. E.g. in electricity and magnetism, we can preserve Maxwell's equations via $\vec{V} \to \vec{-}V$ and $\vec{V}_{axial} \to +\vec{V}_{axial}$.

• In QM, parity is represented by a unitary operator such that $P|\vec{r}\rangle = |-\vec{r}\rangle$ and $P^2 = 1$. So the eigenvectors of P have eigenvalue ± 1. A state is even or odd under parity if $P|\psi\rangle = \pm|\psi\rangle$; in position space, $\psi(-\vec{x}) = \pm\psi(\vec{x})$. In radial coordinates, parity takes $\cos\theta \to -\cos\theta$ and $\phi \to \phi + \pi$, so $P|\alpha, \ell, m\rangle = (-1)^\ell|\alpha, \ell, m\rangle$. This is clear also from the $\ell = 1$ case, any getting general ℓ from tensor products.

If $[H, P] = 0$, then the non-degenerate energy eigenstates can be written as P eigenstates.

Parity in 1d: $PxP = -x$ and $PpP = -p$. The Hamiltonian respects parity if $V(-x) = V(x)$. The energy eigenstates can thus be chosen to be even or odd. E.g. 1d particle in a box. E.g. for 1d, SHO the state $|n\rangle$ has parity $(-1)^n$, since the a and $a^\dagger$ operators are parity odd. Similarly for the 3d SHO: the state with quantum numbers $|n_x, n_y, n_z\rangle$ has parity $(-1)^{n_x+n_y+n_z}$. This agrees with the fact that the parity is $(-1)^\ell$, since $\ell = n$ mod 2.

Recall example of bound states in a potential well, $V(x) = -V_0\theta(\frac{1}{2}a - |x|)$. Write down even and odd parity solutions, and the conditions from having ψ and ψ' continuous. Even parity solutions thus have $p\tan(pa/2\hbar) = \hbar\kappa$ with $p \equiv \sqrt{(2m(E + |V|)}$ and $\hbar\kappa = \sqrt{-2mE}$, where $E < 0$. Odd parity solutions have $p\cot(pa/2\hbar) = -\hbar\kappa$. For a shallow V_0, only one even solution, then get alternating even / odd solutions as V_0 increases.

E.g. symmetric double well, or SHO-type double well: $E_A > E_S$. Double well example (mention $e^{-S_{Euc}/\hbar}$ tunneling and instantons).

• Parity selection rules: if $[H, P] = 0$, time evolution preserves parity and the states and operators can be assigned definite parity. Show that $\langle\chi|\mathcal{O}|\psi\rangle = 0$ unless $(-1)^{P_\chi + P_\mathcal{O} + P_\psi} = 1$, where $(-1)^{P_{\chi,\psi,\mathcal{O}}}$ are the parity signs of the operators and states. Illustrate with SHO examples.