2/2/18 Physics 212b Homework 3, due Friday Feb 9

1. Consider a 1d particle in an infinite square well, centered at the origin. Let $|n\rangle$ be an energy eigenstate. Consider the correlator $\langle n|x^r p^s|m\rangle$ (where x and p denote the operators) for integers n, m, r, s and write the selection rules for which ones must vanish by P symmetry. Also write the selection rules for which ones must vanish by T symmetry. Now write an integral expression for the correlator in position space, and argue for the above selection rules directly in terms of the integral.

2. Same as the previous question, but for the simple harmonic oscillator. Give the selection rules for the correlator $\langle n|x^r p^s|m\rangle$ that follow from P symmetry. Also those from T symmetry. Now argue for these selection rules by writing x and p in terms of creation and annihilation operators. You don't need to bother evaluating the correlator, just use properties of a and $a^\dagger$ to note that you indeed get zero when the selection rule says it's zero.

3. A deuteron has spin 1. Use the Wigner-Eckart theorem to find the ratios of the expectation values of the electric quadrupole moment operator $Q^{\ell=2}_{m=0}$ for the three orientations of the deuteron ($m = -1, 0, 1$).

4. A charged particle with spin operator $\vec{S}$ has an electric dipole moment operator $\mu\vec{S}$, so H contains the interaction term $-\mu\vec{S}\cdot\vec{E}$. Show that this violates both parity and time reversal if the particle is in a spherically symmetric electrostatic potential $\phi(r)$, even when no external electric field is present.

5. Consider a Hamiltonian for a spin 1 system that is given by

$$H = AS_z^2 + B(S_x^2 - S_y^2).$$

Solve this exactly to find the normalized energy eigenstates and eigenvalues. Is H invariant under time reversal? How do the normalized eigenstates that you found transform under time reversal?

6. Calculate the three lowest energy levels, together with their degeneracies, for the following systems:

(a) Three non-interacting, non-identical spin 1/2 particles in a 3d box of length L.

(b) Four non-interacting, identical spin 1/2 particles in a 3d box of length L.

7. A beam of particles of spin s is scattered by a spin-dependent potential. Let $\vec{p}_{in}$ be the initial momentum, and the particles scattered into a certain direction are observed with a detector. Let $\vec{p}_{out}$ be the momentum of these particles and D be the scattering plane defined by $\vec{p}_{in}$ and $\vec{p}_{out}$. Suppose that the incident particles are non-polarized, so their spin state is represented by a density operator $\rho_{in} = 1/(2s+1)$.

(a) Show that, if the interaction is invariant under rotation and parity, then the density operator ρ_{out} representing the spin state of the scattered particles is symmetrical with respect to the plane D.

(b) Show that the density operator representing the spin state of a particle of spin 1/2 can be expressed in the form $\rho = \frac{1}{2}(1 + \vec{P} \cdot \vec{\sigma})$, where $\vec{\sigma}$ are the Pauli matrices and $\vec{P}$ is a vector of length between 0 and 1, which defines the state of polarization of the particle.

(c) Taking the incident unpolarized particles to have spin 1/2, argue that the vector $\vec{P}_{out}$ defining the polarization of the scattered particles must be perpendicular to the scattering plane.