Physics 225b, Homework 6 / take home final, Due Friday March 24.

1. Estimate the Kerr parameter a for the Earth.

2. Work out the range of angular velocity Ω_{obs} for an observer inside the ergosphere at fixed r. Show that this range becomes increasingly limited as the observer is located closer to the horizon, and is eventually limited to the single value Ω_H.

3. de Sitter space ($\Lambda > 0$) has metric

$$ds^2 = -H(r)dt^2 + H^{-1}dr^2 + r^2 d\Omega^2, \qquad H(r) = 1 - \frac{\Lambda}{3}r^2.$$

(a) Draw the Penrose diagram of de Sitter space and draw orbits (as arrows, showing the direction) of the Killing vector $K = \partial_t$.

(b) Find the surface gravity κ (see Mar 1 lecture notes) at the Killing horizon where $K = \partial_t$ is null. Also, find the area A of the horizon.

(c) Write the Euclidean version of de Sitter space by taking $t \to i\tau$ and show that a coordinate transformation can be made to make the Euclidean metric regular at the horizon so long as τ is periodic. Find the periodicity.

(d) The Schwarzschild de Sitter metric is as above, with $H = 1 - (2GM/r) - \frac{\Lambda}{3}r^2$. Verify that for $9\Lambda M^2 < 1$ there are two horizons. These are interpreted as that of the black hole and that of de Sitter space, and the inequality shows that there is a maximum M that can fit in de Sitter space before the black hole fills the deSitter horizon. Interesting fact: increasing M decreases the area of the outer horizon.

4. According to the LIGO press release, their event was interpreted as gravity waves from the merger of two black holes, of masses $29M_\odot$ and $36M_\odot$, with $3M_\odot$ converted into gravitational waves during the merger. Compute the entropy change ΔS_{BH} from the merger. Note: the final state black hole is rotating, but feel free to neglect rotation contributions here.

5. Show that the surface gravity of the event horizon of a Kerr black hole of mass M and angular momentum J is

$$\kappa = \frac{\sqrt{M^4 - J^2}}{2M(M^2 + \sqrt{M^4 - J^2})}.$$

6. Show that the area of the event horizon of a Kerr-Newman black hole is

$$A = 8\pi[M^2 - \tfrac{1}{2}Q^2 + \sqrt{M^4 - Q^2M^2 - J^2}].$$