Physics 225b, Homework 5, Due Wednesday March 1.

Exercises all based on those in Hartle, chapters 9 and 12.

1. Two particles fall radially in from infinity in the Schwarzschild geometry. One starts with $e = 1$ and the other starts with $e = 2$. A stationary observer at $r = 6GM$ measures the speed of each when they pass by. How much faster is the second particle moving at that point?

2. A spaceship is moving without power in a circular orbit about a black hole of mass M, at a Schwarzschild radius of $r = 7GM$. What is the period of the orbit as measured by an observer at infinity? What is the period as measured by a clock in the spaceship?

3. An observer falls feet first into a Schwarzschild black hole looking down at her feet. Is there ever a moment when she cannot see her feet? Can she see her feet when her head is crossing the horizon? If so, what radius does she see them at? Does she ever see her feet hit the singularity at $r = 0$ assuming that she remains intact until her head hits that radius? Analyze these questions with an Eddington-Finkelstein or Kruskal diagram.

4. Two observers in two rockets hover above a Schwarzschild black hole of mass M, at fixed radius r such that $e^{r/2GM}\sqrt{-1 + R/2GM} = \frac{1}{2}$ at fixed angular position. The first observer leaves this position at $t = 0$ and travels into the black hole on a straight line in a Kruskal diagram, until destroyed in the singularity at the point where it crosses the line $X = 0$. The other observer continues to hover at r. Sketch the worldlines of the two observers on a Kruskal diagram. Is the observer who goes into the black hole following a timelike world line? What is the latest Schwarzschild time after the first observer departs that the other observer can send a light signal that will reach the first before being destroyed in the singularity?

5. If light is emitted by a collapsing star, with frequency ω_* when the light is emitted at $r_E \approx 2GM$, then the frequency observed by a receiver is $\omega_R \propto \omega_* e^{-t_R/4GM}$, where t_R is the time that it takes the light to get to the receiver. Derive this (coordinate independent) result using Kruskal coordinates.