Physics 225b, Homework 3, Due Monday Feb 6.

1. Two objects of mass M have a head-on collision at event $(0,0,0,0)$. At $t=\infty$ they were at $x=\pm\infty$ with zero velocity.

   (a) Using Newtonian theory show that their position for $t\leq 0$ are $x(t)=\pm Ct^{2/3}$ and find the constant C.

   (b) For what separations is the Newtonian approximation reasonable?

   (c) Find $h^{TT}_{xx}(t)$ and $(x,y,z)=(0,R,0)$

   [Carroll]

2. Fill in the details to verify that the metrics given in lecture ds^2 for de Sitter and anti-de Sitter indeed follow from the induced metric on the surfaces in one higher dimension. Also, verify that their Riemann tensor satisfies $R_{\rho\sigma\mu\nu}=(R/12)(g_{\rho\mu}g_{\sigma\nu}-g_{\rho\nu}g_{\sigma\mu})$ with R a constant. Relate R to the radius C of the hyperboloid surface.

3. Consider de Sitter space in coordinates where

$$ds^2=-dt^2+e^{Ht}(dx^2+dy^2+dz^2).$$

   Solve the geodesic equation for co-moving observers (x^i =constant) to find the affine parameter as a function of t. Show that the geodesics reach $t=-\infty$ in a finite affine parameter, demonstrating that these coordinates fail to cover the entire space-time. [Carroll]

4. Consider the 2d space-time with metric

$$ds^2=-x^2dt^2+dx^2$$

   where $x\geq 0$.

   (a) Find the shape $x(t)$ of null geodesics.

   (b) Write the equation to determine the shape of time-like geodesics.

   (c) Verify that the coordinate change $x=(X^2-T^2)^{1/2}$, $t=\tanh^{-1}(T/X)$ leads to $ds^2=-dT^2+dX^2$, i.e. 2d Minkowski space.