Physics 225b, Homework 2, Due Monday January 30.

1. Consider a large, thin shell of matter, with mass M and radius R, slowly rotating with angular velocity Ω. [From Carroll and Wald]

(0) Find $T^{\mu\nu}$ to order v^1. As discussed in class, the only non-zero components to this order are T^{0i}. It is essentially similar to finding $\vec{J}$ for a rotating, uniformly charged spherical shell, and the following parts are similar to computing $\vec{E}$ and $\vec{B}$ in that case. (If interested, you can peek at http://journals.aps.org/pr/pdf/10.1103/PhysRev.143.1011 for some additional details.)

(a) Verify that $\vec{G}$ vanishes inside the shell, and calculate $\vec{H}$.

(b) Calculate the rotation (relative to the background metric $\eta_{\mu\nu}$ of a freely falling observer sitting at the center of the shell; i.e. the precession of the spatial components of a parallel-transported vector at the center of the shell.

2. Show that the Lorentz gauge condition $\partial_\mu \bar{h}^{\mu\nu} = 0$ is equivalent to $\partial^2 x^\mu = 0$. [Carroll]

3. Consider the metric

$$ds^2 = -(dudv + dvdu) + a^2(u)dx^2 + b^2(u)dy^2,$$

(a) Calculate the Christoffels and the Riemann tensor. (b) Show that Einstein's equations in vacuum is satisfied if $a(u)$ and $b(u)$ satisfy some equations. (c) Show that an exact solution can be found with $a(u)$ and $b(u)$ given in terms of an arbitrary function $f(u)$. [Carroll]

4. Gravitational waves can be detected by monitoring the distance between two free flying masses. If one of these masses is equipped with a laser and accurate clock, and the other with a good mirror, the distance between the masses can be measured by timing how long it takes for a pulse of laser light to make the round-trip journey. How would you want your detector oriented to register the largest response from a plane wave of the form

$$ds^2 = -dt^2 + (1 + A\cos(\omega(t-y)))dx^2 + dy^2 + (1 - A\cos(\omega(t-y)))dz^2?$$

If the masses have mean separation L, what is the largest change in the arrival time of the pulses caused by the wave? What frequencies would go undetected? [Carroll]