2/18/16 Homework 4. Due Feb 29

1. For $\lambda\phi^4$, we have $m_B^2 = Z_m m_R^2$ (check your lecture notes for the notation). Define $Z_m \equiv 1 + \sum_k c_k(\lambda)\epsilon^{-k}$. To two loops, i.e. to order $\hat{\lambda}^2$, where $\hat{\lambda} \equiv \lambda/16\pi^2$, one computes
$$Z_m = 1 + \epsilon^{-1}(\hat{\lambda} - \frac{5}{12}\hat{\lambda}^2) + \epsilon^{-2}2\hat{\lambda}^2$$
Verify that the coefficient c_2 of the $1/\epsilon^2$ term is completely determined by c_1 and the condition that γ_m have a smooth $\epsilon \to 0$ limit. Verify that the c_1 and c_2 given above satisfy this relation (using the expression given in class for $\beta(\lambda, \epsilon)$ to one loop).

2. Consider the theory
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 - h\phi\bar{\psi}\psi.$$
Compute the 1-loop 1PI propagator $\Pi'(p^2)$ for the scalar ϕ, including both the ϕ loop that we have already discussed, and also the new contribution from the Fermion loop. Use dimensional regularization and minimal subtraction for the counter terms. You should find that there is now a 1-loop contribution to the δZ_ϕ counterterm.

3. Same theory as question 2, compute the 1-loop 1PI propagator for the Fermion ψ. Use dim. reg. and minimal subtraction, canceling the $1/\epsilon$s using counterterms for the Fermion mass and δZ_ψ. You should find that there is a 1-loop contribution to the δZ_ψ counterterm.

4. Using the results from the previous two questions, compute the 1-loop anomalous dimensions γ_ϕ and γ_ψ. Verify that both are positive (as is required by unitarity for gauge invariant quantities).