1/6/15 Ken Intriligator's Phys 4D Lecture outline

⋆ Reading: Volume 2, chapter 18, of Feynman Lectures (see link on class website).

• Recall waves on strings, and the 1d wave equation. Traveling wave solutions, e.g. $\psi = A\cos(kx - \omega t)$, with $k = 2\pi/\lambda$, $\omega = 2\pi/T$, and $v = \omega/k = \lambda/T$.

Now the 3d wave equation. Example of sound waves. Solution examples, e.g. plane wave $s = s_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$ and spherical wave. Emphasize linearity and superposition of solutions. Example of constructive and destructive interference for sound waves. Sound intensity and superposition. We'll see that light has similar properties.

• On to Maxwell's equations in vacuum, and electromagnetic waves. Recall first Maxwell's equations including sources (in both SI and Gaussian units):

$$\vec{\nabla}\cdot\vec{E}_{SI} = \rho_{SI}/\epsilon_0, \qquad \nabla\cdot\vec{E}_G = 4\pi\rho_G, \tag{1}$$

$$\vec{\nabla}\times\vec{B}_{SI} - \mu_0\epsilon_0\frac{\partial\vec{E}_{SI}}{\partial t} = \mu_0\vec{J}_{SI}, \qquad \vec{\nabla}\times\vec{B}_G - \frac{1}{c}\frac{\partial\vec{E}_G}{\partial t} = \frac{4\pi}{c}\vec{J}_G, \tag{2}$$

$$\vec{\nabla}\cdot\vec{B}_{SI} = 0, \qquad \nabla\cdot\vec{B}_G = 0, \tag{3}$$

$$\vec{\nabla}\times\vec{E}_{SI} + \frac{\partial\vec{B}_{SI}}{\partial t} = 0, \qquad \vec{\nabla}\times\vec{E}_G + \frac{1}{c}\frac{\partial\vec{B}_G}{\partial t} = 0 \tag{4}$$

You probably learned the SI (MKS) version (LHS). Back when I was a kid, physicists mostly learned the Gaussian version (RHS), so it's more natural for me. The thing that I like about the Gaussian (CGS) version is, as we can immediately see, $\vec{E}_G$ and $\vec{B}_G$ have the same units. This will be nice when we discuss relativity, since relativistic transformations turn out to electric and magnetic fields into each other. The first two equations say how electric charges and currents source $\vec{E}$ and $\vec{B}$. (I heard from your 4C instructor that the $\partial_t\vec{E}$ term in (2) wasn't discussed, for lack of time.) The second pair of equations is similar to the first pair, with $\vec{E}\leftrightarrow -\vec{B}$, except that there are 0's instead of ρ and $\vec{J}$ type source terms. This is because, as far as we know, there are no magnetic monopoles. (It's not theory-forbidden, just an observational fact that there don't seem to be any around here.)

• In empty space, $\rho = \vec{J} = 0$. We can combine the equations to show that we get the wave equation for $\vec{E}$ and $\vec{B}$, showing that there are traveling waves of $\vec{E}$ and $\vec{B}$, moving at $v = c = (\mu_0\epsilon_0)^{-1/2} = 2.99792458\times 10^8 m/s$ (actually, this is the modern definition of the meter)

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0, \qquad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = 0, \tag{5}$$

The Maxwell equations involving the curl relate $\vec{E}$ and $\vec{B}$ to each other – if you know one, you can solve for the other. The upshot, as you will verify in a HW assignment, is that electromagnetic waves have a direction of propagation, given by wavenumber vector $\vec{k}$ with $|\vec{k}| = 2\pi/\lambda$ the wavenumber, and $\vec{k} \cdot \vec{E} = \vec{k} \cdot \vec{B} = 0$ and $\vec{E} \times \vec{B} \sim \vec{S} \sim \vec{k}$.