2/11/13 Lecture outline

• Relativity for $v \ll c$ (we'll do general case a bit later). Galilean relativity, $t' \approx t$, $\vec{r}' \approx \vec{r} - \vec{v}_r t$. Find that $\vec{E}$ and $\vec{B}$ in the $'$ frame are:

$$\vec{E}' \approx \vec{E} + \vec{v}_r \times \vec{B}/c, \qquad \vec{B}' \approx \vec{B} - \vec{v}_r \times \vec{E}/c.$$

The sources are modified as (note $\vec{v}' \approx \vec{v} - \vec{v}_r$)

$$\rho' \approx \rho, \qquad \vec{J}' \approx \vec{J} - \rho \vec{v}_r.$$

• Check the force law:

$$\rho \vec{E} + \frac{1}{c} \vec{J} \times \vec{B} \approx \rho' \vec{E}' + \frac{1}{c} \vec{J}' \times \vec{B}'.$$

two sides differ at $O(v_r^2/c^2)$. Example: consider charge q moving with velocity $\vec{v}$, in external $\vec{E}$ and $\vec{B}$. In charge's rest frame only electric force, so $\vec{E}' \approx \vec{E} + \vec{v} \times \vec{B}/c$.

Can check Maxwell's equations in frames, e.g. convert $\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$ to rocket frame using $\frac{\partial}{\partial t}|_{\vec{r}} = \frac{\partial}{\partial t'}|_{\vec{r}'} - \vec{v}_r \cdot \nabla'$, $\nabla = \nabla'$, fits with $\vec{B}' \approx \vec{B} - \vec{v} \times \vec{E}/c$. Also charge conservation in rocket frame. Everything here to $O(v_r^2/c^2)$. Of course, Maxwell's equations are already fully relativistic, as we'll discuss more soon.

• Maxwell's equations in vacuum, $\rho = \vec{J} = 0$. Get the wave equation for $\vec{E}$ and $\vec{B}$: $\partial^2 \vec{E} = \partial^2 \vec{B} = 0$, where $\partial^2 \equiv \frac{1}{c}^2 \frac{\partial^2}{\partial t^2} - \nabla^2$.

• Plane wave solutions: $\vec{E} = Re\vec{E}_0 \exp(i\vec{k}\cdot\vec{x} - \omega t)$, $\vec{B} = \widehat{k} \times \vec{E}$, $\vec{E}_0 \cdot \vec{k} = 0$. $\vec{S} = cE^2 \widehat{k}/4\pi$. Plug into $T_{ij} = \frac{1}{4\pi}\left(\frac{1}{2}(E^2 + B^2)\delta_{ij} - E_i E_j - B_i B_j\right)$, get $T_{ij} = \frac{E^2}{4\pi}\widehat{k}_i \widehat{k}_j$ (light pressure).

Fourier expansion ($\phi = 0$ gauge, all sums over $\vec{k}$ and $-\vec{k}$)

$$\vec{A}(\vec{r}, t) = \sum_{\vec{k}} \vec{A}_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{r}},$$

with $A_{\vec{k}}(t) = a_{\vec{k}} e^{-ickt} + a^*_{-\vec{k}} e^{ikct} = A^*_{-\vec{k}}$ to get $\vec{A}^* = A$. Then,

$$\vec{E} = -\frac{1}{c}\partial_t \vec{A} = \sum_{\vec{k}} \vec{E}_k(t) e^{i\vec{k}\cdot\vec{r}}$$

with $\vec{E}_{\vec{k}} = -\frac{1}{c}\dot{\vec{A}}_{\vec{k}}(t) = ik(\vec{a}_{\vec{k}} e^{-ickt} - \vec{a}^*_{-\vec{k}} e^{ikct}) = \vec{E}^*_{-\vec{k}}$, which gives $\vec{E}^* = \vec{E}$, and

$$\vec{B} = \nabla \times \vec{A} = \sum_k i\vec{k} \times A_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{r}},$$

$$U = \frac{V}{4\pi}\sum_k k^2|a_k|^2, \qquad \vec{P}_{field} = \frac{V}{4\pi}\sum_k k^2\widehat{k}|a_k|^2.$$

• Consider the wave equation $\partial^2\psi(\vec{r},t)$, which the components of $\vec{E}$ and $\vec{B}$ satisfy. Taking $\psi = Ref(\vec{r})e^{-ickt}$, get $(\nabla^2+k^2)f(\vec{r}) = 0$. An example is the plane wave, $f = e^{-\vec{k}\cdot\vec{r}}$. Now consider a spherical wave solution. Show

$$(\nabla^2 + k^2)\frac{e^{ikr}}{r} = \frac{1}{r^2}\frac{d}{dr}(r^2\frac{d}{dr}\frac{e^{ikr}}{r}) = -4\pi\delta(\vec{r}).$$