1/7/13 Lecture outline

⋆ Garg chapters1+2

• Observe that interactions not instantaneous. Implies fastest communication speed c. Principle of special relativity: results of physics experiments the same in all inertial frames. We might discuss this in more detail here a bit later (TBD).

Consequence: need to introduce $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$. Field theory. Fields are real and physical, not mathematical artifacts. They carry momentum, energy, angular momentum.

• Facts: elementary particles can be assigned various physical characteristics: mass, electric charge, spin, ... Conserved quantities: energy, momentum, angular momentum, and electric charge. Electric charges just add (or subtract), $Q = \sum_n q_n$, charges can be positive or negative. (Unlike gravity, where masses additive). Since opposite charges attract, charges tend to neutralize – why gravity usually dominates over E&M on large distance scales. Also, charge quantization, in integer units of the charge of the electron.

• Two aspects of electrodynamics: (i) how test (small) charges are affected by $\vec{E}$ and $\vec{B}$; (ii) how charges make $\vec{E}$ and $\vec{B}$. Aspect (i) is simply given by the Lorentz force law, or equivalently we can write it in terms of Lagrangians, once we introduce the scalar and vector potential, ϕ and $\vec{A}$. Aspect (ii) will occupy us for most of the course.

• (i) $\vec{F} = q(\vec{E} + c^{-1}\vec{v}\times\vec{B})$. (ii) Maxwell's equations, in Gaussian units

$$\begin{aligned}\nabla\cdot\vec{E} &= 4\pi\rho \\ \nabla\times\vec{B} - \frac{1}{c}\partial_t\vec{E} &= \frac{4\pi}{c}\vec{J}\end{aligned} \tag{1}$$

$$\begin{aligned}\nabla\cdot\vec{B} &= 0 \\ \nabla\times\vec{E} + \frac{1}{c}\partial_t\vec{B} &= 0.\end{aligned} \tag{2}$$

Mention SI units soon.

• Behavior under $P : \vec{x}\to-\vec{x}$ and $T : t\to -t$.

• Charge conservation: $\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0$. With charged particles,

$$\rho(\vec{x},t) = \sum_n q_n\delta(\vec{x}-\vec{x}_n(t)), \qquad \vec{J}(\vec{x},t) = \sum_n q_n\dot{\vec{x}}_n\delta(\vec{x}-\vec{x}_n(t)).$$

• Maxwell's eqns are fully relativistic: led to relativity. $J^\mu = (c\rho,\vec{J})$ is a 4-vector, i.e.it transforms like $x^\mu = (ct,\vec{x})$ between inertial frames. Equations (1) transform as a 4-vector, and so does (2).

The above are linear: superposition! Compare and contrast with other forces. Linearity related to the neutrality of the force carrier, the photon. Expt: $q_\gamma < 10^{-30} q_e$. Also, photon is massless. Expt: $m_\gamma < 10^{-24} m_e$. Only two polarizations. Why we can see distant stars. Why $F \sim q_1 q_2 / r^2$, so at an interior point inside a uniformly charged (not necessarily spherical) shell get cancellation of forces from the charges on opposite sides (Ben Franklin): $q_2 \sim r^2 \Omega$. So $F \sim q_1 r^2 r^{-2} (\Omega - \Omega) = 0$. Works only for massless photon. Aside: the photon is massive inside a superconductor – why they have such bizarre properties!

• Maxwell's equations (1) show how electric charges affect $\vec{E}$ and $\vec{B}$. Maxwell's equations (2) would have been analogous, for magnetic charges, but the zeros on the RHS express the absence of magnetic charges. We could have written $4\pi \rho_{mag}$ and $4\pi \vec{J}_{mag}/c$ on their RHS, if there are magnetic monopoles. Comment on magnetic monopoles. Electric magnetic duality.

• Not having magnetic sources simplifies life, though. Can solve (2) via

$$\vec{E} = -\nabla \phi - c^{-1} \partial_t \vec{A}, \qquad \vec{B} = \nabla \times \vec{A}. \tag{3}$$

$(\phi, \vec{A})$ transforms as a 4-vector, just like $(ct, \vec{x})$, under Lorentz transformations.

• Gauge invariance. $\vec{A} \to \vec{A} + \nabla f$, $\phi \to \phi - c^{-1} \partial_t f$ is a symmetry for any $f(t, \vec{x})$. Becomes of fundamental importance in quantum field theory. Gauge symmetries directly connected with forces.

• $L = L_0 + \frac{q}{c} \dot{\vec{x}} \cdot \vec{A} - q\phi$. Gives $\vec{p} = \vec{p}_0 + \frac{q}{c} \vec{A}$. Gauge invariance of EOM: $L \to L + \frac{q}{c} \frac{df}{dt}$. In QM we replace $\vec{p} \to \frac{\hbar}{i} \nabla$. The wavefunction changes under a gauge transformation $\psi \to e^{\frac{iqf}{\hbar c}} \psi$. Preserves probabilities, $|\psi|^2$. Deep fact: physics is gauge invariant.

• $q = n q_e$, so $f \sim f + 2\pi R$, with $R = \hbar c / q_e$. In KK theory, this is the 5-th dimensional circle. Aside on Dirac quantization and monopoles, maybe later.

• Back to non-speculative, brass tacks. Unit conversions. Compare Coulomb force, energy density, and Lorentz force in Gaussian (CGS) vs SI (MKS)

$$\frac{q_{Gau}^2}{r^2} = \frac{q_{SI}^2}{4\pi \epsilon_0 r^2} \tag{4}$$

$$\frac{1}{8\pi} (E_{Gau}^2 + B_{Gau}^2) = \tfrac{1}{2} (\epsilon_0 E_{SI}^2 + \mu_0^{-1} B_{SI}^2) \tag{5}$$

$$q_{Gau} (\vec{E}_{Gau} + \frac{1}{c} \vec{v} \times \vec{B}_{Gau}) = q_{SI} (\vec{E}_{SI} + \vec{v} \times \vec{B}_{SI}). \tag{6}$$

So

$$q_{SI} = \sqrt{4\pi \epsilon_0} q_{Gau}, \quad \vec{E}_{SI} = \vec{E}_{Gau} / \sqrt{4\pi \epsilon_0}, \quad \vec{B}_{SI} = \sqrt{\frac{\mu_0}{4\pi}} \vec{B}_{Gau}, \quad c = 1/\sqrt{\mu_0 \epsilon_0}. \tag{7}$$

Example: $B_{wire} = 2I / c r_\perp = \mu_0 I / 2\pi r_\perp$. Units: $[q] = M^{1/2} L^{3/2} T^{-1}$, $[E] = M^{1/2} L^{-1/2} T^{-1} = [B_{Gau}]$. Next time: electrostatics in vacuum (Garg chapter 3).