203a Homework 3, due Jan. 23

1. Show that the functions

$$\rho(\mathbf{x}, t) = qf(\mathbf{x} - \mathbf{x}_p(t))$$

$$\mathbf{j}(\mathbf{x}, t) = q\frac{d\mathbf{x}_p}{dt}f(\mathbf{x} - \mathbf{x}_p(t))$$

satisfy the charge-conservation continuity equation, where $\mathbf{x}_p(t)$ is an arbitrary function of t, and f is another arbitrary function.

2. Suppose that there is a surface current density, $\vec{J} = \vec{K}\delta(z)$, with $\vec{K} = K\hat{y}$ and K is a constant. Find $\vec{B}$ everywhere two ways: (i) using Stoke's and Gauss' law and symmetries; (ii) by direct integration using the general formula.

3. Consider a particle with Lagrangian $L(\vec{x}, \vec{v}) = -mc^2\sqrt{1 - v^2/c^2} + \frac{q}{c}\vec{A}(\vec{x}) \cdot \vec{v} - q\phi(\vec{x})$.

   (a) Find the momentum $\vec{p} = \partial L/\partial \vec{v}$.

   (b) Write the Euler-Lagrange equations and verify that they give

$$\frac{d}{dt}(\gamma m\vec{v}) = q\vec{E} + \frac{q}{v}\vec{v} \times \vec{B}$$

with $\gamma \equiv 1/\sqrt{1 - v^2/c^2}$ and $\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial A}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$.

   (c) Compute $H = \vec{p} \cdot \vec{v} - L$ and show $H = \gamma mc^2 + q\phi$. Write out $H = H(\vec{p}, \vec{x})$.

4. Garg 31.2.

5. Garg 31.4.