203a Homework 2, due Jan. 23

1. Just work it out.

2. Revenge of the 4π.

$$\nabla^2(r^{-k}) = C\delta^D(\vec{r}),$$

Note using the analog of the previous question that in D-dimensional spherical coordinates $\nabla f(r) = \widehat{r}\frac{d}{dr}f(r)$ and $\nabla \cdot (\widehat{r}F_r(r)) = r^{-(D-1)}\frac{d}{dr}(r^{D-1}F_r(r))$, so $\nabla^2 \cdot f(r) = r^{-(D-1)}\frac{d}{dr}(r^{D-1}\frac{d}{dr}f(r))$. Clearly then we want $k = D-2$. This is also clear from dimensional analysis. Gauss law tells us $C = \int_{\partial V}(2-D)R^{-D-1}da = (2-D)\Omega$ where $\Omega = A/R^{D-1}$ is the total solid angle.

3. Garg 22.1 (give details, more than just the answers stated in the book).

Use $\vec{B}(\vec{x}) = \frac{I}{c}\int d\vec{\ell'} \times (\vec{x}-\vec{x}')/|\vec{x}-\vec{x}'|^3$ and cylindrical coordinates: $\vec{x} = z\widehat{z}$, $\vec{x}' = R\widehat{\rho}'$ and $d\vec{\ell'} = R\widehat{\phi}'d\phi'$, so $|\vec{x}-\vec{x}'| = \sqrt{z^2+R^2}$. It's clear from symmetry that $\vec{B}$ will point on the $\widehat{z}$ axis, and indeed $\widehat{\phi}' \times \widehat{\rho}' = -\widehat{z}$, whereas the term $\sim d\vec{\ell'} \times \vec{x} \sim \widehat{\phi}' \times \vec{z} \sim \widehat{\rho}'$ integrates to zero. We then immediately get the answer given in the book (with $R = a$).

4. Garg 23.4

Work it out from the previous result, using superposition.

5. Garg 27.1.

Consider just the dipole term, with one coil giving $\vec{m} = \widehat{z}IA/c$ and the other giving $-\vec{m}$. Put $\vec{m}$ at $a\widehat{z}$ and the $-\vec{m}$ at the origin:

$$\vec{A}(\vec{r}) \approx \frac{IA}{c}\left(\frac{\widehat{z}\times(\vec{r}-a\widehat{z})}{|\vec{r}-a\widehat{z}|^3} - \frac{\widehat{z}\times\vec{r}}{r^3}\right).$$

Taylor expand it out, keeping just the term linear in a. (This gives the book's hint.)