203a Homework 2, due Jan. 23

1. Review of vector calculus in different coordinate systems (e.g. spherical, or cylindrical, but here more generally). The coordinates are ξ_i, and their associated orthonormal unit vectors are $\widehat{e}_i$ and the length element is $d\vec{\ell} = \sum_{i=1}^{3} h_i(\xi) d\xi_i \widehat{e}_i$, for some general functions $h_i(\xi)$. You can loop up the answers to the below in many places, but the idea of this **exercise** is to draw pictures and make the results your own, so you can re-derive them if you ever need them.
   (a) Write the expressions for the area element $d\vec{a}$ and volume element dV.
   (b) Write the expression for ∇f, such that $\int_1^2 \nabla f \cdot d\vec{\ell} = f(2) - f(1)$.
   (c) Write the expression for $\nabla \cdot \vec{F}$, such that Gauss' law holds.
   (d) Write the expression for $\nabla \times \vec{F}$ such that Stoke's law holds.

2. Revenge of the 4π. Imagine that you are Luke Skywalker, or Princess Leia (your choice!), and you want become a 4π-master (first step to being a Jedi master!). Yoda kindly pushes you through a portal, to another world where there are D space dimensions. He now tortures ... ahem, *strengthens* you with the following exercise: *find the Green's function of the Laplacian in this D dimensional world.* Specifically, find the values of k and C such that

$$\nabla^2(r^{-k}) = C\delta^D(\vec{r}),$$

   where r is the radial coordinate. Hint: Gauss' law $\int_V dV\, \nabla \cdot \vec{F} = \int_{\partial V} \vec{F} \cdot d\vec{a}$ still holds in this other world, where now dV has units $Length^D$ and $d\vec{a}$ has units $Length^{D-1}$. You also might enjoy knowing that the volume and surface area of a sphere in this dimension is

$$V = \frac{\pi^{D/2}}{(D/2)!} R^D, \qquad A = \frac{2\pi^{D/2}}{((D/2)-1)!} R^{D-1},$$

   where recall e.g. $(-\frac{1}{2})! = \Gamma(\frac{1}{2}) = \sqrt{\pi}$. Write k and C for general D. As a special case, you should recover our old friends for $D = 3$: $k_{D=3} = 1$, and $C_{D=3} = -4\pi$. Yes!

3. Garg 22.1 (give details, more than just the answers stated in the book).

4. Garg 23.4

5. Garg 27.1.