203a Homework 6, due March 12

Only turn in the $*$ problems

1. Verify that $\mathcal{L}_\theta = \theta \vec{E} \cdot \vec{B}$ are total derivative terms, so don't affect the Euler-Lagrange equations of motion. Specifically, verify that $\vec{E} \cdot \vec{B} = \nabla \cdot (\nabla\phi \times \vec{A} - \frac{1}{2}\vec{A} \times \frac{1}{c}\partial_t \vec{A}) - \frac{1}{2c}\partial_t(\vec{A} \cdot \nabla \times \vec{A})$.

2*. Using $\int \vec{E} \cdot d\vec{a} = 4\pi Q_{encl}$ and $\oint \vec{B} \cdot d\vec{\ell} = \frac{4\pi}{c} I_{encl}$, together with symmetry, determine $\vec{E}$ and $\vec{B}$ in terms of $\rho = \rho(r)$ and $\vec{J} = \hat{z} J_z(r) + \hat{\theta} J_\theta(r)$, where (r, θ, z) are cylindrical coordinates.

3. Jackson 1.1.

4. Jackson 1.4.

5. Jackson 1.6.

6*. Jackson 1.8.

7. Jackson 5.6.

8*. A parallel plate capacitor consists of two circular disks of radius R separated by $d \ll R$. It is slowly charging at the rate $I = \dot{Q}$. Calculate $\vec{E}$ and $\vec{B}$ inside.

9. Two parallel wires have separation d and carry charge per length λ in the lab frame. They are at rest in the lab frame, with zero current.

(a) Calculate the force per length between the wires in a $'$ frame that moves parallel to the wires with velocity v. Do this by finding λ' and $\vec{J}'$ and using them to compute $\vec{E}'$ and $\vec{B}'$ and the associated force.

(b) Compute the same quantity as in part (a) by instead computing the force in the lab frame, and Lorentz transforming it to the lab frame. Use the fact that f^μ is a 4-vector and $d^2 r_\perp = d^2 r'_\perp$.