1/15/10 Lecture 4 outline

⋆ Reading: Srednicki ch. 9 and 10.

• Last time:

$$Z_{free}[J] = Z_0[J] = \exp(-\tfrac{1}{2}\hbar \int d^4x d^4y J(x) D_F(x-y) J(y)), \tag{1}$$

with

$$D_F(x-y) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik(x-y)}}{k^2 - m^2 + i\epsilon},$$

and, including interactions,

$$Z[J] = N \exp[\frac{i}{\hbar} S_{int}[-i\frac{\delta}{\delta J}]) Z_{free}[J], \tag{2}$$

where N is an irrelevant normalization factor (independent of J). Correspondingly, the green's functions are given by

$$\begin{aligned} G^{(n)}(x_1 \dots x_n) &= \frac{\int [d\phi] \phi(x_1) \dots \phi(x_n) \exp(\frac{i}{\hbar} S_I[\phi]) \exp[\frac{i}{\hbar} S_{free}]}{\int [d\phi] \exp(\frac{i}{\hbar} S_I[\phi]) \exp[\frac{i}{\hbar} S_{free}]}, \\ &= \frac{1}{Z[J]} \prod_{j=1}^{n} \left(-i\hbar \frac{\delta}{\delta J(x_j)} \right) \cdot Z[J] \big|_{J=0}. \end{aligned}$$

(The denominator (in both lines) cancels off the vacuum bubble diagrams, which don't depend specifically on the Green's function.)

• Illustrate the above formulae, and relation to Feynman diagrams, e.g. $G^{(1)}$, $G^{(2)}$ and $G^{(4)}$ in $\lambda\phi^4$ theory. The functional integral accounts for all the Feynman diagrammer; even symmetry factors etc. come out simply from the derivatives w.r.t. the sources, and the expanding the exponentials. E.g.

$$G^{(1)}(x_1) = \frac{1}{Z[J]} (-i \frac{\delta}{\delta J(x_1)}) \sum_{N=1}^{\infty} \frac{1}{N!} \left(-i \frac{\lambda}{4!} \int d^4y (-i)^4 \frac{\delta^4}{\delta J(y)^4} \right)^N Z_0[J] \big|_{J=0}.$$

etc.

Illustrate $Z[J]$ computation of various $G^{(n)}$ for $V_{int} = \frac{\lambda_4}{4!}\phi^4$ theory, connecting to the diagrams. (Srednicki discusses this for the $V_{int} = \frac{\lambda_3}{3!}\phi^3$ theory example.)

Consider, for example, the 4-point function $G^{(4)}(x_1, x_2, x_3, x_4) \equiv \langle T\phi(x_1) \dots \phi(x_4) \rangle / \langle 0|0 \rangle$ in $\frac{\lambda_4}{4!}\phi^4$. So take 4-fuctional derivatives w.r.t. the source, at points $x_1 \dots x_4$, i.e. $\delta/\delta J(x_1) \dots \delta/\delta J(x_4)$. The $\mathcal{O}(\lambda^0)$ term thus comes from expanding the exponent in (1) to quadratic order (4 J's), corresponding to the disconnected diagrams with two propagators.

Each propagator ends on a point x_i. This is like the 4-point function in the SHO homework. Now consider the $\mathcal{O}(\lambda)$ contribution, coming from expanding out the interaction part of the exponent in (2) to $\mathcal{O}(\lambda)$. There are now 4 extra $\delta/\delta J(y)$, for a total of 8, so the contributing term comes from expanding the exponent in (1) to 4-th order, i.e. there are 4 propagators. This gives the connected term, along with several disconnected terms. Go through these terms and their combinatorics.