2/26/10 Lecture 16 outline

• New topic, functional integral for fermions and gauge fields. Path integral of same, general form. Need to understand some new issues with integrations. Fermions first. Grassmann number integrals, $\int d\theta(A+B\theta)=B$. Complex θ, θ^*, $\int d\theta^* d\theta \exp(-\theta^* b\theta)=b$. $\prod_i \int d\theta_i^* d\theta_i \exp(-\theta_i^* B_{ij}\theta_j)=\det B$. $\prod_i \int d\theta_i^* d\theta_i \exp(-\theta_i^* B_{ij}\theta_j)\theta_k\theta_l^* = (B^{-1})_{kl}\det B$.

• We can introduce sources for the fields:

$$Z[\bar\eta_i,\eta_i] = \int d\bar\theta_i d\theta_i \exp(i(A_{ij}\bar\theta_i\theta_j + \bar\eta_i\theta_i + \bar\theta_i\eta_i])$$
$$= \int d\bar\theta_i d\theta_i (1+i(\bar\theta, A\theta))(1+i\bar\eta\theta)(1+i\bar\theta\eta),$$
$$= -i\det A \exp(-i\bar\eta_i A^{-1}_{ij}\eta_j).$$

• Generalize to functional integrals over fermionic fields;

$$Z[\bar\eta,\eta] = \int [d\bar\psi][d\psi]\exp(i\int d^4x[\bar\psi(i\partial\!\!\!/ - m)\psi + \bar\eta\psi + \bar\psi\eta]$$
$$= Z_0 \exp[-\int d^4x d^4y \bar\eta(x) S_F(x-y)\eta(y).$$

where

$$S_F[x-y] = i(i\partial\!\!\!/ - m)^{-1} = \int \frac{d^4k}{(2\pi)^4}\frac{ie^{-ik(x-y)}}{k\!\!\!/ - m + i\epsilon}.$$

Get e.g.

$$\langle 0|T\psi(x)\bar\psi(y)|0\rangle = Z_0^{-1}(-i\frac{\delta}{\delta\bar\eta(x)})(i\frac{\delta}{\delta\eta(y)})Z[\eta,\bar\eta]|_{\eta,\bar\eta=0} = S_F(x-y).$$

Gives the Feynman rules for fermions that we discussed last quarter.

• For fermions, the $\det B$ is in the numerator, whereas for scalars it's in the denominator. The functional integral gives e^{iW}. So the sign of the contribution to W is opposite for closed scalar vs fermion loops: every closed fermion loop gets an extra -1 factor. (This relative minus sign is put to good use with supersymmetry!)

• Now gauge fields. Important point: gauge invariance.