2/24/10 Lecture 15 outline

• Last time:

$$\left(\frac{\partial}{\partial \ln \mu} + \beta(\lambda_R)\frac{\partial}{\partial \lambda_R} + \gamma_m m_R \frac{\partial}{\partial \ln m_R} - n\gamma\right)\tilde{\Gamma}_R^{(n)}(p_1, \dots p_n; \lambda_R, m_R, \mu) = 0$$

Here

$$\beta(\lambda) \equiv \frac{d}{d\ln \mu}\lambda_R$$

$$\gamma = \tfrac{1}{2}\frac{d}{d\ln \mu}\ln Z_\phi$$

$$\gamma_m = \frac{d\ln m_R}{d\ln \mu}.$$

We find for $\lambda\phi^4$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3).$$

Integrating, this gives

$$\lambda = \lambda_0\left(1 - \frac{3}{16\pi^3}\lambda_0 \ln(\mu/\mu_0)\right)^{-1}.$$

The running coupling becomes small in the IR, and blows up at a "Landau pole in the UV". "Triviality."

We also found $\gamma_\phi \geq 0$, which is true for any gauge invariant field, where $\gamma_\phi = 0$ iff it is a free field. This follows from the spectral decomposition result that $Z \leq 1$.

• Note: $\beta > 0$ means the coupling is small in the IR, and large in the UV. Such theories are "not asymptotically free" or are "IR free." Most theories are like this, e.g. $\lambda\phi^4$ (e.g. the Higgs coupling), QED, Yukawa interactions. Picture for QED of vacuum polarization, screening the bare charge. If $\beta < 0$, then the coupling is small in the UV, and large in the IR. Such theories are "asymptotically free;" only non-Abelian gauge theories, like QCD, are like that. Means vacuum anti-screens charges.

• QED: one loop beta function, $\beta(e) = e^3/12\pi^2$, leads to $\alpha_{eff}(\mu)^{-1} = \alpha_0^{-1} - \frac{1}{6\pi}\log(\mu/\mu_0)$. QCD: one loop beta function $\beta(g) = -Cg^3/2$, leads to $g^{-2}(\mu) = g_0^{-2} + C\log(\mu/\mu_0)$. Discuss the interpretation.

• Pictures of RG flows. Briefly outline GUT idea and unification of the running couplings.