1/27/09 Lecture 8 outline

• It is useful to define a further specialization of the diagrams, those that are 1PI: one - particle irreducible. The definition is that the diagrams is connected, and moreover remains connected upon removing any one internal progagator (and amputating all external legs).

•Examples of $n = 2, 4, 6$ point 1PI diagrams in $\lambda\phi^4$.

• In momentum space, it is defined from the 1PI diagram, with all external momenta taken to be incoming:

$$\text{1PI diagram} \equiv i\tilde{\Gamma}^{(n)}(p_1, \ldots p_n),$$

where the external propagators are amputated, and the $(2\pi)^4\delta^4(\sum_i p_i)$ is omitted. If there is an interaction like $g\phi^n/n!$, then, at tree-level, $\tilde{\Gamma}^{(n)} = g$. Special definition for case $n = 2 :$ we define the 1PI diagram to be $-i\Pi'(p)$, and we instead define

$$i\tilde{\Gamma}^{(2)}(p, -p) = \text{1PI diagram} + i(p^2 - m^2) = i(p^2 - m^2 - \Pi'(p^2)).$$

Define position space 1PI diagrams by Fourier transform. They correspond to

$$\Gamma^{(n)}(x_1, \ldots x_n) = \langle T\phi(x_1)\ldots\phi(x_n)\rangle|_{1PI}.$$

• 2-point function, via summing geometric series:

$$D(p) = \frac{i}{\tilde{\Gamma}^{(2)}} = \frac{i}{p^2 - m^2 - \Pi'(p^2)}.$$

$-i\Pi'$ is computed from the 1PI diagrams. $\Pi'(p^2)$ is called the self-energy, like momentum dependent mass term. The special definition of $\tilde{\Gamma}^{(2)}$ is because $D(p) = i/\tilde{\Gamma}^{(2)}$ will be nice, and allow extending to higher point functions.

• The point of the 1PI diagrams is that the quantum loop corrections are simply obtained by replacing the vertices with the 1PI greens functions! Indeed, Draw pictures for $n = 2, 4, 6$ point functions. Obtain the full $W[J]$ via tree-graphs assembled from the 1PI building blocks.