1/15/09 Lecture 5 outline

• Last time:

$$Z_{free}[J] = Z_0[J] = \exp(-\tfrac{1}{2}\hbar \int d^4x d^4y J(x) D_F(x-y) J(y)),$$

with

$$D_F(x-y) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik(x-y)}}{k^2 - m^2 + i\epsilon},$$

and, including interactions,

$$Z[J] = N \exp[\frac{i}{\hbar} S_{int}[-i\frac{\delta}{\delta J}]) Z_{free}[J],$$

where N is an irrelevant normalization factor (independent of J). Correspondingly, the green's functions are given by

$$G^{(n)}(x_1 \dots x_n) = \frac{\int [d\phi]\phi(x_1)\dots\phi(x_n)\exp(\frac{i}{\hbar}S_I[\phi])\exp[\frac{i}{\hbar}S_{free}]}{\int [d\phi]\exp(\frac{i}{\hbar}S_I[\phi])\exp[\frac{i}{\hbar}S_{free}]},$$
$$= \frac{1}{Z[J]} \prod_{j=1}^n \left(-i\hbar \frac{\delta}{\delta J(x_j)}\right) \cdot Z[J]\big|_{J=0}.$$

(The denominator (in both lines) cancels off the vacuum bubble diagrams, which don't depend specifically on the Green's function.)

• Illustrate the above formulae, and relation to Feynman diagrams, e.g. $G^{(1)}$, $G^{(2)}$ and $G^{(4)}$ in $\lambda\phi^4$ theory. The functional integral accounts for all the Feynman diagrammer; even symmetry factors etc. come out simply from the derivatives w.r.t. the sources, and the expanding the exponentials. E.g.

$$G^{(1)}(x_1) = \frac{1}{Z[J]}(-i\frac{\delta}{\delta J(x_1)}) \sum_{N=1}^{\infty} \frac{1}{N!}\left(-i\frac{\lambda}{4!}\int d^4y (-i)^4 \frac{\delta^4}{\delta J(y)^4}\right)^N Z_0[J]\big|_{J=0}.$$

etc.

Illustrate $Z[J]$ computation of various $G^{(n)}$ for $V_{int} = \frac{\lambda_3}{3!}\phi^3$ theory, connecting to the diagrams.