2/5/09 Lecture 10 outline

• • One-loop effective potential for $\lambda\phi^4$. The effective potential is found from $\Gamma[\phi]$, keeping the terms with no derivatives. Find

$$V_1(\phi) = i\sum_{n=1}^{\infty}\frac{1}{2n}\int\frac{d^4k}{(2\pi)^4}\left(\lambda\frac{1}{k^2-m^2+i\epsilon}\frac{\phi^2}{2}\right)^n$$
$$= \tfrac{1}{2}\int\frac{d^4k_E}{(2\pi)^4}\ln\left(1+\frac{\frac{1}{2}\lambda\phi^2}{k_E^2+m^2}\right)$$

(S. Coleman and E. Weinberg.) Symmetry factors: $1/n!$ not all the way cancelled, because of Z_n rotation symmetry, and reflection, gives $1/2n$. At each vertex, can exchange external lines, so $1/4!$ not all the way cancelled, leads to $1/2$ for each vertex. Still have to explain how to handle k_E integral.

• Let's consider the 1-loop term in $\tilde{\Gamma}^{(2)}$ for $\lambda\phi^4$. Get

$$-i\Pi'(p^2) = (-i\lambda)\tfrac{1}{2}\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2} + \text{more loops.}$$

Now rotate to Euclidean space, $d^4k = id^4k_E$,

$$\Pi'(p^2) = \tfrac{1}{2}\lambda\int\frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2+m^2} + \text{more loops.}$$

Recall expression $\Omega_{D-1} = 2\pi^{D/2}/\Gamma(D/2)$ is the surface area of a unit sphere S^{D-1}. For $D=4$, get $\Omega_3 = 2\pi^2$, so

$$\Pi'(p^2) = \frac{\lambda m^2}{32\pi^2}\int_0^{\Lambda^2/m^2}\frac{udu}{u+1} = \frac{\lambda m^2}{32\pi^2}\left(\frac{\Lambda^2}{m^2} - \log(1+\frac{\Lambda^2}{m^2})\right).$$

Here Λ is a UV momentum cutoff. Result is quadratically (and also log) divergent as $\Lambda\to\infty$. The subject of renormalization is the physical interpretation of these divergences. The first thing to do is to regulate them, as we did above with a momentum cutoff. There are other ways to regulate. How one regulates is physically irrelevant. The physics is in the renormalization interpretation of the regulated results, and at the end of the day the choice of regulator doesn't matter.