Homework 3, Due Feb. 28, 2008

1. For a general supersymmetric Lagrangian, $\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}) + \int d^2\theta W(\Phi) + h.c.$, the equations of motion (Euler-Lagrange equations) can be written directly in superspace as

$$D^2\left(\frac{\partial K}{\partial \bar{\Phi}^i}\right) + \frac{\partial \bar{W}}{\partial \bar{\Phi}^i} = 0,$$

$$\bar{D}^2\left(\frac{\partial K}{\partial \Phi^i}\right) + \frac{\partial W}{\partial \Phi^i} = 0.$$

Verify this in components for the case of a single chiral superfield, with $K = \Phi\bar{\Phi}$, for general $W(\Phi)$, as follows:

(a) Write out the Lagrangian in components, and write out the Euler Lagrange equations for ϕ, and ψ_α, and F.

(b) Expand out the above superspace equations of motion, and verify that they agree.

2. Compute the number of supersymmetric vacua for the following examples:

(a) $K = X\bar{X}$, $W = \frac{1}{2}mX^2$.

(b) $K = X\bar{X}$, $W = fX$.

(c) $K = \sqrt{X\bar{X}}$, $W = fX$.

(d) $K = X\bar{X}$, $W = fX + \epsilon X^2$.