2/5/07 Lecture 8 outline

• Last time:

$$W[J] = \Gamma[\overline{\phi}] + \int d^4x J(x)\overline{\phi}(x).$$

$$\Gamma[\overline{\phi}] = W[J] - \int d^4x J(x)\overline{\phi}(x).$$

$\overline{\phi}(x)$ can be interpreted as the average of $\phi(x)$ in the presence of the source; sometimes called classical field:

$$\overline{\phi}(x) = \frac{\delta W[J]}{\delta J(x)} = \frac{\langle 0|\phi(x)|0\rangle_J}{\langle 0|0\rangle_J}.$$

The functional derivatives of $\Gamma[\overline{\phi}]$, upon setting $\overline{\phi} = 0$, give $\Gamma^{(n)}(x_1, \ldots x_n)$ • Recall $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \phi J$, with the source term J. The classical field EOM is

$$(\partial_\mu\partial^\mu + m^2)\phi_c = -\frac{1}{3!}\lambda\phi_c^3 + J(x).$$

As discussed last time, we can solve this in perturbation theory in λ, with only tree-level diagrams. The generating functional for tree-level diagrams is $W_c[J] = S[\phi_c] + \int d^4x J\phi_c$.

Again,

$$\Gamma[\phi] = \frac{1}{\hbar}\left(S[\phi] + \mathcal{O}(\hbar)\right).$$

The field $\overline{\phi}$ satisfies the same equation, up to order $\hbar$ corrections:

$$(\partial_\mu\partial^\mu + m^2)\overline{\phi} = -\frac{1}{3!}\lambda\overline{\phi}^3 + J(x) + \mathcal{O}(\hbar).$$

So, at the classical level, $\phi_c = \overline{\phi}$. But $\overline{\phi}$ includes the quantum loop corrections.

• Draw the pictures of the loop diagrams, as included in tree-level diagrams, assembled from the 1PI building blocks.

• One-loop effective potential for $\lambda\phi^4$. The effective potential is found from $\Gamma[\phi]$, keeping the terms with no derivatives. Find

$$V_1(\phi) = i\sum_{n=1}^{\infty}\frac{1}{2n}\int\frac{d^4k}{(2\pi)^4}\left(\lambda\frac{1}{k^2 - m^2 + i\epsilon}\frac{\phi^2}{2}\right)^n$$
$$= \tfrac{1}{2}\int\frac{d^4k_E}{(2\pi)^4}\ln\left(1 + \frac{\frac{1}{2}\lambda\phi^2}{k_E^2 + m^2}\right)$$

(S. Coleman and E. Weinberg.) Symmetry factors: $1/n!$ not all the way cancelled, because of Z_n rotation symmetry, and reflection, gives $1/2n$. At each vertex, can exchange external lines, so $1/4!$ not all the way cancelled, leads to 1/2 for each vertex. Still have to explain how to handle k_E integral. We'll come back to this later.