3/12/07 Lecture 18 outline

• Last time, gauge invariance. $\psi_g = e^{ieg(x)}\psi$, $D_\mu = \partial_\mu + ieA^\mu$. Lagrangian $\bar\psi(i\not{D}-m)\psi$ is gauge invariant. $A^T_\mu = P_{\mu\nu}A^\nu$, $P_{\mu\nu} = g_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2$. $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}A^T_\mu \partial^2 g^{\mu\nu}A^T_\nu$. Can't invert kinetic terms uniquely to find Green's function. We need to fix the gauge.

• Do this via

$$1 = \int [d\alpha(x)]\delta(G(A^\alpha)) \det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right) = \Delta\int [d\alpha]\delta(G(A^\alpha)),$$

where $G(A) = 0$ is some gauge fixing condition, e.g. Lorentz gauge, $G(A) = \partial_\mu A^\mu$ and

$$\Delta = \det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right)_{G=0}.$$

Δ is the Faddeev-Popov determinant. Write the functional integral as (using the gauge invariance of measure and action)

$$\int [d\alpha][dA]\Delta \exp(iS[A])\delta(G[A]).$$

Have factored out the integral over the group volume.

Take e.g. $G = \partial^\mu A_\mu - f(x)$ for some function $f(x)$. Then $\Delta \sim \det(\partial^2)$ is a constant. Get

$$e^{iW} = N\int (dA)e^{iS}\delta(\partial^\mu A_\mu - f) = N\int [dA][df]e^{iS}\delta(\partial^\mu A_\mu - f)G(f) = N\int [dA]e^{iS}G(\partial A),$$

for arbitrary functional G. Choose $G(f) = \exp(-\frac{1}{2}i\xi^{-1}\int d^4x f^2)$, for some real number ξ. Get

$$e^{iW} = N\int [dA]\exp(iS - \tfrac{1}{2}\xi^{-1}\int d^4x(\partial^\mu A_\mu)^2).$$

Then get for the propagator

$$D_{\mu\nu} = \frac{-i}{k^2}[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + \xi\frac{k_\mu k_\nu}{k^2}].$$

Popular choices: $\xi = 1$ is Feynman propagator, $\xi = 0$ is Landau gauge propagator. Physics is ξ independent (result of gauge invariance, which yields Ward-Takahashi identities). Let's choose to use Feynman gauge.