3/7/07 Lecture 17 outline

• Last time: $\prod_i \int d\theta_i^* d\theta_i \exp(-\theta_i^* B_{ij}\theta_j) = \det B$. $\prod_i \int d\theta_i^* d\theta_i \exp(-\theta_i^* B_{ij}\theta_j)\theta_k \theta_l^* = (B^{-1})_{kl} \det B$.

• We can introduce sources for the fields:

$$\begin{aligned} Z[\bar\eta_i, \eta_i] &= \int d\bar\theta_i d\theta_i \exp(i(A_{ij}\bar\theta_i\theta_j + \bar\eta_i\theta_i + \bar\theta_i\eta_i]) \\ &= \int d\bar\theta_i d\theta_i (1 + i(\bar\theta, A\theta))(1 + i\bar\eta\theta)(1 + i\bar\theta\eta), \\ &= -i \det A \exp(-i\bar\eta_i A_{ij}^{-1}\eta_j). \end{aligned}$$

• Generalize to functional integrals over fermionic fields;

$$\begin{aligned} Z[\bar\eta, \eta] &= \int [d\bar\psi][d\psi] \exp(i \int d^4x[\bar\psi(i\partial\!\!\!/ - m)\psi + \bar\eta\psi + \bar\psi\eta] \\ &= Z_0 \exp[-\int d^4x d^4y \bar\eta(x) S_F(x-y)\eta(y). \end{aligned}$$

where

$$S_F[x-y] = i(i\partial\!\!\!/ - m)^{-1} = \int \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik(x-y)}}{k\!\!\!/ - m + i\epsilon}.$$

Get e.g.

$$\langle 0|T\psi(x)\bar\psi(y)|0\rangle = Z_0^{-1}(-i\frac{\delta}{\delta\bar\eta(x)})(i\frac{\delta}{\delta\eta(y)})Z[\eta, \bar\eta]|_{\eta,\bar\eta=0} = S_F(x-y).$$

• For fermions, the $\det B$ is in the numerator, whereas for scalars it's in the denominator. The functional integral gives e^{iW}. So the sign of the contribution to W is opposite for closed scalar vs fermion loops: every closed fermion loop gets an extra -1 factor. (This relative minus sign is put to good use with supersymmetry - to find out more, take the topics class next year!)

• Now gauge fields. Important point: gauge invariance. Write $A = A_\mu dx^\mu$. Recall gauge symmetry $A \to A^\alpha = A + d\alpha(x)$, with $\psi \to e^{-ie\alpha(x)}\psi$. Redundancy in description, can only observe gauge invariant quantities. Need to replace $\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu$. Then $D_\mu^\alpha \psi^\alpha = e^{-ie\alpha} D_\mu \psi$ transforms nicely, with just an overall phase, and $\bar\psi D_\mu \psi$ is gauge invariant. So the Dirac lagrangian, $\bar\psi(iD\!\!\!\!/ - m)\psi$ is gauge invariant. In functional integral, will have $\int [dA] \exp(iS)$. Integration measure must be gauge invariant, implies it gets a factor of gauge orbit volume. Would like to integrate only over a slice of inequivalent gauge fields, without integrating over the gauge orbits. Need to do this, since otherwise there is no well defined B^{-1}. Recall $S = \int d^4x[-\frac{1}{4}F_{\mu\nu}^2] = \frac{1}{2}\int d^4k A_\mu(x)(\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu(x)$. Note action vanishes if $\tilde A_\mu(k) = k_\mu \alpha(k)$. Gauge invariance. $A_\mu^T = P_{\mu\nu}A^\nu$, $P_{\mu\nu} = g_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2$. $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}A_\mu^T \partial^2 g^{\mu\nu} A_\nu^T$. Can't invert kinetic terms uniquely to find Green's function. We need to fix the gauge.