3/5/07 Lecture 16 outline

- Last time:

$$\beta(\lambda) \equiv \frac{d}{d\ln\mu}\lambda_R$$

$$\gamma_\phi = \tfrac{1}{2}\frac{d}{d\ln\mu}\ln Z_\phi$$

$$\gamma_m = \frac{d\ln m_R}{d\ln\mu}.$$

$$\beta(\lambda,\epsilon) = -\epsilon\lambda + \beta(\lambda)$$

$$\beta(\lambda) = \lambda^2\frac{da_1}{d\lambda}$$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3).$$

Integrating, this gives

$$\lambda = \lambda_0\left(1 - \frac{3}{16\pi^3}\lambda_0\ln(\mu/\mu_0)\right)^{-1}.$$

We similarly have

$$\gamma_\phi(\lambda,\epsilon) = \tfrac{1}{2}\frac{d}{d\ln\mu}\ln Z_\phi$$

where

$$Z_\phi = 1 + \sum_k Z_\phi^{-k}(\lambda)\epsilon^{-k}.$$

So

$$\gamma_\phi(\lambda,\epsilon) = \tfrac{1}{2}\beta(\lambda,\epsilon)\frac{d}{d\lambda}\ln Z_\phi.$$

Using $\beta(\lambda,\epsilon) = -\epsilon\lambda + \beta(\lambda)$, we get

$$\gamma_\phi = -\tfrac{1}{2}\lambda\frac{d}{d\lambda}Z_\phi^{(1)}.$$

We similarly have $m_B^2 = (m^2 + \delta_{m^2})Z_\phi^{-1} \equiv Z_m m^2$ and

$$\gamma_m(\lambda) = \tfrac{1}{2}\frac{d\ln m^2}{d\ln\mu} = -\tfrac{1}{2}\frac{d\ln Z_m}{d\ln\mu} = -\tfrac{1}{2}\beta\frac{d\ln Z_m}{d\lambda} = \tfrac{1}{2}\lambda\frac{dZ_m^{(1)}}{d\lambda}$$

where $Z_m^{(1)}$ means the coefficient of $1/\epsilon$. In all these cases, only the coefficient of $1/\epsilon$ matters.

In particular, for $\lambda\phi^4$ we have

$$\gamma_\phi = \frac{1}{12}\frac{\lambda^2}{(16\pi^2)^2} + \dots \qquad \gamma_m = \tfrac{1}{2}\frac{\lambda}{16\pi^2} - \frac{5}{12}\frac{\lambda^2}{(16\pi^2)^2} + \dots$$

For any gauge invariant field ϕ, we always have $\gamma_\phi \geq 0$, where $\gamma_\phi = 0$ iff it is a free field. This follows from the spectral decomposition result that $Z \leq 1$.

• Note: $\beta > 0$ means the coupling is small in the IR, and large in the UV. Such theories are "not asymptotically free" or are "IR free." Most theories are like this, e.g. $\lambda\phi^4$ (e.g. the Higgs coupling), QED, Yukawa interactions. Picture for QED of vacuum polarization, screening the bare charge. If $\beta < 0$, then the coupling is small in the UV, and large in the IR. Such theories are "asymptotically free;" only non-Abelian gauge theories, like QCD, are like that. Means vacuum anti-screens charges.

• QED: one loop beta function, $\beta(e) = e^3/12\pi^2$, leads to $\alpha_{eff}(\mu)^{-1} = \alpha_0^{-1} - \frac{1}{6\pi}\log(\mu/\mu_0)$. QCD: one loop beta function $\beta(g) = -Cg^3/2$, leads to $g^{-2}(\mu) = g_0^{-2} + C\log(\mu/\mu_0)$.

• Pictures of RG flows.

• New topic, functional integral for fermions and gauge fields. Path integral of same, general form. Need to understand some new issues with integrations. Fermions first. Grassmann number integrals, $\int d\theta(A + B\theta) = B$. Complex θ, θ^*, $\int d\theta^* d\theta \exp(-\theta^* b\theta) = b$. $\prod_i \int d\theta_i^* d\theta_i \exp(-\theta_i^* B_{ij}\theta_j) = \det B$. $\prod_i \int d\theta_i^* d\theta_i \exp(-\theta_i^* B_{ij}\theta_j)\theta_k\theta_l^* = (B^{-1})_{kl} \det B$.