3/2/07 Lecture 15 outline

- Last time:

$$\left(\frac{\partial}{\partial \ln \mu} + \beta(\lambda_R)\frac{\partial}{\partial \lambda_R} + \gamma_m m_R \frac{\partial}{\partial \ln m_R} - n\gamma\right)\tilde{\Gamma}_R^{(n)}(p_1, \ldots p_n; \lambda_R, m_R, \mu) = 0$$

Here

$$\beta(\lambda) \equiv \frac{d}{d\ln\mu}\lambda_R$$

$$\gamma_\phi = \tfrac{1}{2}\frac{d}{d\ln\mu}\ln Z_\phi$$

$$\gamma_m = \frac{d\ln m_R}{d\ln\mu}.$$

This is the Callan-Symanzik equation. It can be integrated, to relate the renormalized Greens functions at different scales μ and μ'. Let us focus on what β and γ mean.

- Understand what β and γ mean: the bare quantities are some function of the renormalized ones and epsilon. E.g. for $\lambda\phi^4$ in MS we have

$$\lambda_B = \mu^\epsilon Z_\phi^{-2}(\lambda + \delta_\lambda) \equiv \mu^\epsilon \lambda Z_\lambda$$

Let us write

$$Z_\lambda \equiv 1 + \sum_k a_k(\lambda)\epsilon^{-k},$$

where e.g. we found $a_1(\lambda) = +3\lambda/16\pi^2$ to one loop (coming from $\delta_\lambda = 3(\lambda^2/16\pi^2)(1/\epsilon)+$ finite and $\delta_Z = 0+$ finite). The bare parameter λ_B is independent of μ, whereas λ depends on μ, such that the above relation holds. Take $d/d\ln\mu$ of both sides,

$$0 = \epsilon\lambda Z_\lambda + \beta(\lambda,\epsilon)Z_\lambda + \beta(\lambda,\epsilon)\lambda\frac{dZ_\lambda}{d\lambda}.$$

Using the above expansion for Z_λ and requiring that $\beta(\lambda,\epsilon)$ be regular at $\epsilon = 0$, so $\beta(\lambda,\epsilon) = \beta(\lambda) + \sum_n \beta_n \epsilon^n$, gives

$$\beta(\lambda,\epsilon) = -\epsilon\lambda + \beta(\lambda)$$

$$\beta(\lambda) = \lambda^2\frac{da_1}{d\lambda}$$

$$\lambda^2\frac{da_{k+1}}{d\lambda} = \beta(\lambda)\frac{d}{d\lambda}(\lambda a_k).$$

The beta function is determined entirely from a_1. The $a_{k>1}$ are also entirely determined by a_1. In k-th order in perturbation theory, the leading pole goes like $1/\epsilon^k$.

We find for $\lambda\phi^4$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3).$$

Integrating, this gives

$$\lambda = \lambda_0 \left(1 - \frac{3}{16\pi^3}\lambda_0 \ln(\mu/\mu_0)\right)^{-1}.$$