2/7/07 Homework 3. Due Feb 14

1. For a scalar field theory, with $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$, show that the EOM are satisfied, up to a contact term:

$$\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x_\mu}\langle T\phi(x)\phi(y)\rangle + \langle V'(\phi(x))\phi(y)\rangle = \alpha\delta(x-y).$$

To do this problem, consider the functional integral for $W[J]$, and use the invariance of the functional integral under a change of variables. The change of variables is $\phi \to \phi + \epsilon f(x)$, where $f(x)$ is an arbitrary function of x, and ϵ is an infinitesimal parameter (drop terms of order ϵ^2 and higher). Derive in this way the above result (and determine the coefficient α). (Don't worry about the Jacobian in the $[d\phi]$ integration measure - it doesn't contribute.) The source term for ϕ is J, and you can think of $f(x)$ as a source for the EOM.

2. Peskin problem 10.3. Only do the part about the first diagram in (10.31).

3 This problem is **optional**. (It's very similar to problem 1.) Consider $\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - m\phi^*\phi - \frac{1}{4}\phi^2\phi^{*2}$, where ϕ is a complex scalar field. The lagrangian has a symmetry, $\phi \to e^{ia}\phi$, so there is a corresponding conserved current $J^\mu(x)$, with $\partial_\mu J^\mu = 0$. Find $J^\mu(x)$, using the Noether method. Now, in analogy with the above problem, use the functional integral to show that the current is conserved in correlation functions, up to contact terms:

$$\frac{\partial}{\partial x}\langle TJ^\mu(x)\phi(y)\rangle = \beta\delta(x-y).$$

Derive this similarly to problem 1 (and, again determine the constant β).