1/10/07 Homework 1, due Jan 22

1. Consider the harmonic oscillator, $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2$. The time evolution, from position $x = x_a$ at time $t = 0$, to position $x = x_b$ at time $t = T$, is given in the Hamiltonian description by

$$U(x_b, x_a; T) = \langle x_b| \exp(-iHT/\hbar)|x_a\rangle.$$

Compute this object **using the Feynman path integral**

$$U(x_b, x_a; T) = \int [dx] \exp(iS/\hbar),$$

where the integral is over all paths with the prescribed end point boundary conditions. Do this problem the same way as the free particle case outlined in class: chop T into N pieces of size ϵ, and do the Gaussian integrals in sequence. Derive in this way the following answer:

$$U = \left(\frac{m\omega}{2\pi i\hbar \sin \omega T}\right)^{1/2} \exp(iS_{cl}/\hbar),$$

with S_{cl} the action evaluated for the classical path:

$$S_{cl} = \frac{m\omega}{2\sin\omega T}[(x_b^2 + x_a^2)\cos\omega T - 2x_b x_a].$$

Note that, as $\omega \to 0$, U becomes the free-particle expression discussed in class.

2. P& S: 9.2a, 9.2b.