$\mathcal{L} = \int d^4\theta \; K(\Phi_i, \bar{\Phi}_{\bar{i}})$ has susy vacua with arbitrary $\langle \phi_i \rangle \in \Sigma$ with metric

$g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} K(\phi_i, \bar{\phi}_{\bar{i}})$. "Non linear sigma model"

Global $U(1)_R$ symmetry ($\sim$ fermion number)

$\phi_i, \bar{\phi}_i$ = neutral, ψ_i charge -1, $\psi^\dagger_{\bar{i}}$ charge $+1$.

To lift $\langle \phi_i \rangle$ degeneracy, add a scalar potential $V(\phi_i, \bar{\phi}_{\bar{i}})$, e.g. ϕ_i mass terms. In susy theory this comes from adding a Superpotential:

$$\mathcal{L} = \int d^4\theta \; K(\Phi_i, \bar{\Phi}_{\bar{i}}) + \int d^2\theta \, W(\Phi_i) + \int d^2\bar{\theta} \, \bar{W}(\bar{\Phi}_{\bar{i}})$$

susy requires W to be holomorphic: $\dfrac{\partial W}{\partial \bar{\Phi}_{\bar{i}}} = 0$

and $\bar{W}$ to be anti holomorphic $\dfrac{\partial \bar{W}}{\partial \Phi_i} = 0$

in the chiral superfields.

Reality: $\bar{W}(\bar{\Phi}_{\bar{i}}) = \left[W(\Phi)\right]^\dagger$

$$\int d^2\theta W \longrightarrow \partial_i W F^i - \frac{1}{2} \partial_i \partial_j W \psi^i \psi^j$$

F^i EOM: $F^i = \frac{1}{2}\Gamma^i_{jk}\psi^j\psi^k - g^{i\bar{j}}\partial_{\bar{j}}\overline{W}$

get

$$\mathcal{L} = g_{i\bar{j}}\left(-\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} + \bar{\psi}^{\bar{j}}(i\not{D})\psi^i\right)$$

$$+ \frac{1}{4}R_{i\bar{j}k\bar{l}}\,\psi^i\psi^k\bar{\psi}^{\bar{j}}\bar{\psi}^{\bar{l}}$$

$$-\frac{1}{2}(\nabla_i\nabla_j W)\,\psi^i\psi^j + h.c. \leftarrow \text{"Yukawa terms"}$$

$$-V(\phi,\bar{\phi})$$

$V \geq 0$ as expected by susy

with scalar potential $V(\phi,\bar{\phi}) = g^{i\bar{j}}\,\partial_i W\,\partial_{\bar{j}}\overline{W}$.

Supersymmetric vacuum requires $\langle F^i\rangle = 0$

~~[illegible]~~ & $\langle\psi^i\rangle = 0$ & $\therefore$ $\langle g^{i\bar{j}}\partial_{\bar{j}}\overline{W}\rangle = 0$

This gives $\langle V(\phi,\bar{\phi})\rangle = 0$ in a susy vacuum as expected since $\langle H\rangle = 0$ if Q_α & $\bar{Q}_{\dot{\alpha}}$ annihilate the vacuum.

For non degenerate $g_{i\bar{j}}$ the condition for a susy vacuum is $\therefore$ that there is a solution $\langle\phi_i\rangle$ to the eqns $\frac{\partial W}{\partial \phi_i} = 0$

note this is n eqns in n unknowns $\langle\phi_i\rangle$ for $i = 1 \cdots n$.

If $\langle F^i\rangle \neq 0$, susy is broken &

$\psi^i \sim$ massless Goldstino

<u>Examples</u> take one field Φ & $K = \bar{\Phi}\Phi$

1) $W = \lambda \Phi \longrightarrow F = -\lambda^* \neq 0$ susy broken

no sol'n to $\frac{dW}{d\Phi} = 0$ for $\lambda \neq 0$.

$$\mathcal{L} = -\partial_\mu \bar{\phi}\, \partial^\mu \phi - i \bar{\psi}_{\dot{\alpha}}\, \partial^{\dot{\alpha}\alpha} \psi_\alpha - |\lambda|^2$$

free theory with energy offset: $V = |\lambda|^2 \neq 0$ like a cosmological constant. Breaks susy.

$\psi_\alpha \rightarrow$ massless Goldstino $\quad \mathrm{Tr}(-1)^F = 0$.

2) $W = \frac{m}{2}\Phi^2 \longrightarrow$ the following terms in $\mathcal{L}$:

$-W''\psi\psi = -m\psi\psi \leftarrow \psi$ gets mass m

$-|W'|^2 = -m^2|\phi|^2 \leftarrow$ complex scalar ϕ gets mass m

Same mass, as required by susy.

Susy vacuum requires $W' = 0$ i.e. $\langle\phi\rangle = 0$

This vacuum has $\mathrm{Tr}(-1)^F = 1$.

Recall in susy QM with 2 supercharges

$\mathrm{Tr}(-1)^F = \mathrm{sign}(m)$ with m = real.

Here 4 supercharges & $\mathrm{Tr}(-1)^F = \mathrm{sign}(mm^*) = 1$

Since m = complex, can rotate $m \to e^{i\alpha}m$ &

$\mathrm{Tr}(-1)^F$ must be unchanged under continuous deformations

So $\mathrm{Tr}(-1)^F = 1$ for any $m \neq 0$.

3) $W = \frac{m}{2}\Phi^2 + \lambda\Phi = \frac{m}{2}\tilde{\Phi}^2 - \frac{\lambda^2}{2m}$

$\tilde{\Phi} \equiv \Phi + \lambda/m$

Susy vacuum at $\langle\tilde{\Phi}\rangle = 0$

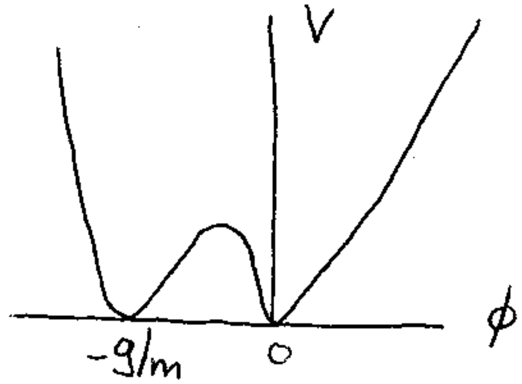
constant term in W has no effect in global susy. (Does contribute in SUGRA)

4) $W = \frac{g}{3}\Phi^3 + \frac{1}{2} m \Phi^2 \longrightarrow$ terms in $\mathcal{L} =$

$-W'' \psi\psi = -(2g\phi + m)\psi\psi \leftarrow$ mass + Yukawa interaction

$-|W'|^2 = -|m\phi + g\phi^2|^2 = -V(\phi, \bar{\phi})$

2 susy vacua : $\phi = 0$ & $\phi = -g/m$



Each vacuum contributes $(-1)^F = 1$, total $\mathrm{Tr}(-1)^F = 2$.

5) $W = g\Phi^{n+1} \longrightarrow$ Susy vacua at $W'(\phi) = 0 \longrightarrow \langle\phi\rangle = 0$. Actually n vacua on top of each other. Deform W

e.g. $W = g\Phi^{n+1} + \lambda\Phi$ or general order $n+1$ polynomial. $W' =$ order n polynomial in complex $\phi \longrightarrow \exists$ n solutions to $W'(\langle\phi\rangle) = 0$. Each has $(-1)^F = 1$ so total $\mathrm{Tr}(-1)^F = n$. n susy vacua.

6) $K = (\bar{\Phi}\Phi)^{1/2}$ $\quad W = \lambda \Phi$

can do change of variables: $X \equiv \Phi^{1/2}$

to get canonical $K = \bar{X} X$ & $W = \lambda X^2$

so must have $\mathrm{Tr}\,(-1)^F = 1$ susy vacuum at $\langle X \rangle = 0$. How to see: $\Phi = \cdots + \theta^2 F$

so $\int d^4\theta K \longrightarrow \cdots + \frac{\partial^2 K}{\partial\phi\partial\bar{\phi}} F\bar{F} = \frac{(\phi\bar{\phi})^{-1/2}}{4} F\bar{F}$

$\int d^2\theta W \rightarrow \lambda F$. So F EOM $\longrightarrow$

$\bar{F} = 4\lambda(\phi\bar{\phi})^{1/2}$ & $V(\phi,\bar{\phi}) = |F|^2 = 16|\lambda|^2 |\phi|^2$

can satisfy $\langle F \rangle = 0$ & $\langle V \rangle = 0$ at $\phi = 0$ ✓

place where $\frac{\partial^2 K}{\partial\phi\partial\bar{\phi}}$ coeff of kinetic terms is singular. Singularity $\Rightarrow$ should change variables to nonsingular field X to see correct physics.

More comments on $U(1)_R$ symmetries. Assign θ^α charge $+1$ & $\bar{\theta}^{\dot{\alpha}}$ charge -1

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

If Φ has charge q, ϕ charge $= q$, ψ charge $= q-1$, F charge $= q-2$.

$$\mathcal{L} = \int d^4\theta\, K + \int d^2\theta\, W + h.c.$$

respects $U(1)_R$ provided K charge $= 0$ & $U(1)_R$ charge of $W = 2$.

E.g. $K = \bar{\Phi}\Phi$ & $W = g\,\Phi^{n+1}$ respects $U(1)_R$ with Φ charge $= 2/n+1$. Adding other terms to W would break this $U(1)_R$

e.g. $W = \frac{g}{3}\Phi^3 + \frac{m}{2}\Phi^2$ has no unbroken $U(1)_R$ if both $g \neq 0$ & $m \neq 0$.

We can pretend that this breaking is "spontaneous" rather than explicit by thinking of g & m as constant expectation values of chiral superfields rather than as parameters. E.g. $W = \frac{g}{3}\Phi^3 + \frac{m}{2}\Phi^2$

	$U(1)_R$ charge
Φ	$2/3$
g	0
m	$2/3$

$\leftarrow$ "field" m is charged under $U(1)_R$. Its vev $\langle m\rangle \neq 0$ $\therefore$ spontaneously breaks $U(1)_R$.

This viewpoint will be very important shortly.

Another comment about $U(1)_R$: R charge of $W = 2$. Scaling dimension of $W = 3$. In a scale inv. susy theory (= superconformal field theory) there is a conserved $U(1)_R$ such that all chiral superfields Φ have

Scaling dim $[\Phi] = \frac{3}{2}$ R charge $[\Phi]$.

Another example: two chiral superfields Φ_1 & Φ_2 with $K = \bar{\Phi}_1 \Phi_1 + \bar{\Phi}_2 \Phi_2$ & $W = \frac{1}{2} g \Phi_1 \Phi_2^2$.

$$V(\phi_i, \bar{\phi}_{\bar{i}}) = \left|\frac{\partial W}{\partial \phi_1}\right|^2 + \left|\frac{\partial W}{\partial \phi_2}\right|^2 = \left|\tfrac{1}{2} g \phi_2^2\right|^2 + |g \phi_1 \phi_2|^2.$$

Susy vacua when $\frac{\partial W}{\partial \phi_1} = 0$ & $\frac{\partial W}{\partial \phi_2} = 0$

$\longrightarrow \langle \phi_2 \rangle = 0 \qquad \langle \phi_1 \rangle =$ arbitrary.

There is a (classical) "moduli" space of supersymmetric vacua for any $\langle \phi_1 \rangle \in \mathbb{C}$.

For $\langle \phi_1 \rangle \neq 0$, the superpotential for Φ_2 is $W \approx \frac{1}{2} g \langle \phi_1 \rangle \Phi_2^2 \longrightarrow$ mass for Φ_2 $= g\langle \phi_1 \rangle$. Since Φ_2 is massive, we can "integrate it out" for energies below its mass. Do path integral $\int [d\Phi_2]$ to get effective field theory for Φ_1.

Low energy quantum effective theory for Φ_1

$$\mathcal{L}_{eff} = \int d^4\theta \; K_{eff}(\Phi_1, \bar{\Phi}_1) + \int d^2\theta \; W_{eff}(\Phi_1) + h.c.$$

Since $\langle \Phi_2 \rangle = 0$, $\quad W_{cl}(\Phi_1) = 0$

so there is the classical moduli space of vacua mentioned earlier. Is this classical degeneracy lifted by quantum effects? I.e. is a $W_{eff}(\Phi_1) \neq 0$ generated?

No! Original theory has $U(1)_R$ symmetry with Φ_1 = neutral & Φ_2 charge = 1

$\langle \phi_1 \rangle \neq 0$ leaves this symmetry unbroken.

So any $W_{eff}(g, \Phi_1)$ must respect this $U(1)_R$ i.e. have charge 2. But can not make charge 2 W_{eff} from neutral g & Φ_1. $\therefore$ $W_{exact}(g, \Phi_1) = 0$.

There is an <u>exact</u> quantum moduli space of degenerate susy vacua $\langle \Phi_1 \rangle$ = arb.

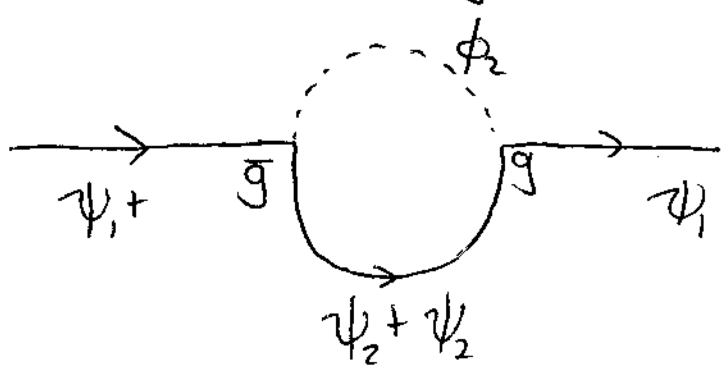
$$\mathcal{L}_{eff} = \int d^4\theta \, K_{eff}(\Phi_1, \bar{\Phi}_1) + \int d^2\theta \, \cancel{W_{eff}}(\Phi_1) + h.c.$$

0

nontrivial quantum effects in here

e.g. $W_{cl}(\Phi_1, \Phi_2) = \frac{1}{2} g \Phi_1 \Phi_2^2$ leads to

Yukawa interactions $\quad -g\,\phi_2 \psi_1 \psi_2 - g\,\phi_1 \psi_2 \psi_2 + h.c.$

get 

renormalizes ψ_1 kinetic term

$$\psi_1^+ (i\partial) \psi_1 \longrightarrow Z^2 \psi_1^+ (i\partial) \psi_1$$

$\psi_1 \to Z \psi_1$ wavefn renormalization. Likewise for scalars. Entire superfield renormalizes as

$\Phi_1 \to Z \Phi_1$ i.e. $\int d^4\theta \, \bar{\Phi}_1 \Phi_1 \longrightarrow \int d^4\theta \, Z^2 \bar{\Phi}_1 \Phi_1$

evaluate above diag $\sim \bar{g} g \int d^4k \frac{1}{(\not{k} - m)} \frac{1}{((p-k)^2 - m^2)}$

extract term $\sim \not{p}$ so $\int \frac{d^4k}{k^4}$ log diverg.

$$\text{get} \quad Z = \overset{\text{tree}}{1} + g\bar{g} \overset{\text{1 loop}}{\ln} \left|\frac{\mu}{m}\right|^2 + \cdots$$

with $m = \langle \phi_1 \rangle$

$$\text{so} \quad K_{eff} = \bar{\Phi}_1 \Phi_1 \Big(\underset{\text{tree}}{1} + \underset{\text{1 loop}}{g\bar{g} \ln\left(\frac{\mu^2}{\bar{\Phi}_1 \Phi_1}\right)} \Big) + \underset{\text{higher order}}{\cdots}$$

General feature

$$\mathcal{L}_{cl} = \int d^4\theta\, K_{cl} + \int d^2\theta\, W_{cl} + h.c.$$

$$\xrightarrow{\text{quantum effective action}} \quad \mathcal{L}_{eff} = \int d^4\theta\, K_{eff} + \int d^2\theta\, W_{eff} + h.c.$$

generally K_{eff} gets complicated corrections in perturbation theory.

But generally W_{eff} gets <u>no renormalization</u> in perturbation theory. Only possible W renormalization is via <u>non-perturbative</u> effects.