Susy

<u>What</u>: bose ⟷ fermi symmetry. Conserved charge Q_s in spinor rep of Lorentz gp, like a fermion.

$$[Q_s, H] = 0 \implies \begin{array}{l} Q_s(\text{boson}) = \text{fermion} \\ Q_s(\text{fermion}) = \text{boson} \end{array}$$

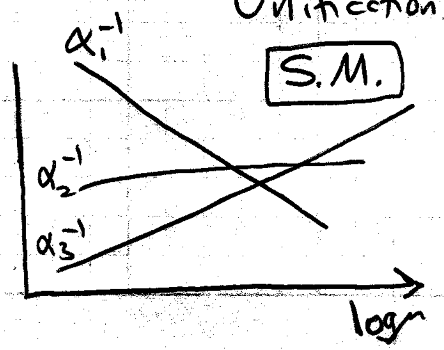
related states have same mass.

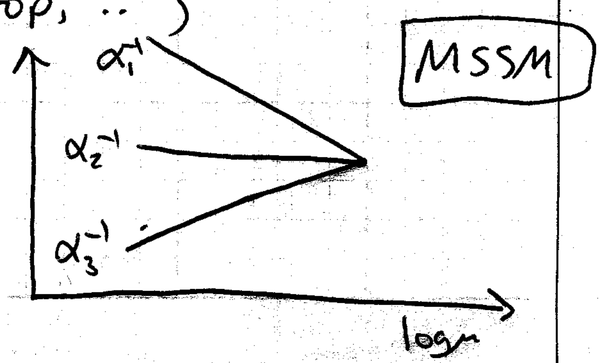
bose → bosino   fermi → sfermi

not observed → must be <u>broken</u> symmetry if it indeed occurs at all in nature.

Still useful.

<u>Why</u>:
- Unique nontrivial extension of Poincare symm of spacetime
- superstrings
- can help with fine tuning problems
- can use to get <u>exact</u> results, gain insight into dynamics of QFT.
- some observational hints (coupling const unification, heavy top, ..)





Symmetries (global or local ⟷ force) & conservation laws.

Conserved current J_μ. $\partial^\mu J_\mu = 0$ ⟶ conserved charge $Q = \int d^3x\, J_0$. $[Q, H] = 0$.

Poincare currents: $T_{\mu\nu}$, $T_{\mu\nu} X_\sigma - T_{\mu\sigma} X_\nu$

charges $P_\mu = \int d^3x\, T_{\mu 0}$, $M_{\nu\sigma} = \int d^3x\, (T_{0[\nu} X_{\sigma]})$

these are tensors whereas other conserved charges e.g. electric charge, baryon number, isospin etc. are all Lorentz scalars. Don't extend Poincare symmetry group. Can we extend Poincare?

☆ <u>Coleman Mandula Thm</u>: Above $d=2$ no interacting theory could have any add'l (bosonic) conserved charges $Q_{\mu\nu\ldots}$ which are not Lorentz scalars. Only P_μ, $M_{\mu\nu}$, & scalar Q conserved charges. Detailed proof. Qualitative reason: consider 2 body scattering 2 in → 2 out. P_μ, $M_{\mu\nu}$ cons → only scattering θ undet'd.

Conservation of other $Q_{\mu \ldots}$ would $\Rightarrow$ other products of momentum conserved $\Rightarrow$ determine scattering $\measuredangle$. Analyticity in scattering angle then $\Rightarrow$ amplitude vanishes for all angles, free theory.

Supersymmetry is the only possible loophole, since the charge transforms as a spinor (fermion) rather than boson. Supercurrent $J_{s\mu}$ & charge $Q_s = \int d^3x \, J_{s0}$

↑ Lorentz spinor index

Susy algebra: $\{Q_s, Q_{s'}\} = \Gamma^{\mu}_{ss'} P_\mu$

e.g. in 3+1d 4 spinor components (like $e^-\uparrow$, $e^-\downarrow$, $e^+\uparrow$, $e^+\downarrow$)

~~For simplicity first consider~~

☆ <u>Superstrings</u>: have supersymmetry in 10d at very high energies ~ M_P. Could persist down to some scale M_s where susy is broken. E.g. photino mass ~ M_s. ~~How~~ how low is M_s?

Fine tuning problems : $S = \int d^4x \sqrt{\det g}\, \mathcal{L}$

$$\mathcal{L} = \Lambda 1 + (\partial_\mu \phi)(\partial^\mu \phi) - \tfrac{1}{2} m^2 \phi^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi + \ldots$$

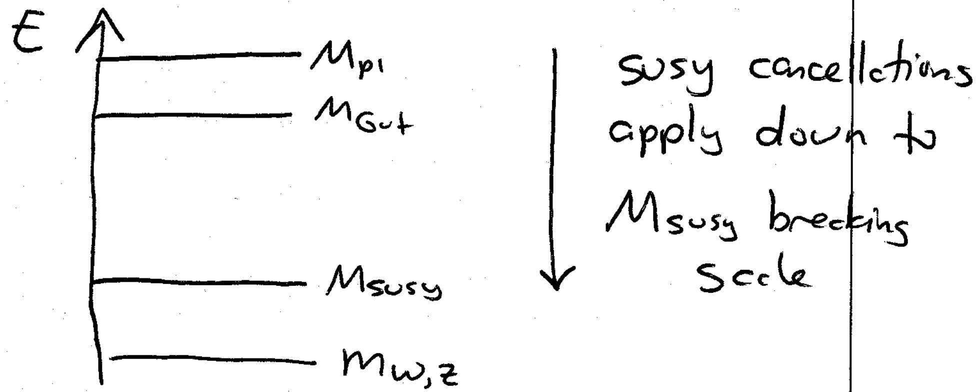
Quantum effects : loops of new particles (beyond S.M.) of mass M induce corrections

$$\delta\Lambda \sim O(1)\, M^4 \qquad \delta m^2_{boson} \sim O(1)\, M^2$$

e.g. $M = M_{pl} \sim 10^{19}$ GeV or $M_{gut} \sim 10^{16}$ GeV

but $\dfrac{\Lambda}{M_{pl}^4} \sim 10^{-120}$ $\qquad \dfrac{m_W^2}{M_{pl}^2} \sim 10^{-34}$

fine tuning problems. Susy can help : bose & fermi quantum effects tend to mostly cancel. Often exactly cancel to all orders in pert. thy. Contributions to masses could be nonperturbatively small e.g. $\sim e^{-c/g^2}$. Intuitive reason : if $m_{fermion} = 0$, extra approximate symmetry $\psi \to e^{i\alpha\gamma_5}\psi$. Preserved by quantum effects up to e^{-c/g^2} effects. Since susy relates boson to fermion, it can keep e.g. $M_{W^\pm}$, M_Z small.



For M_{susy} low enough (e.g. ~ 1 TeV) can help with fine tuning problems. Still need more work to explain why $\frac{\Lambda}{M_{susy}^4} \ll 1$.

☆ Exact Results: Susy is a symmetry, makes QFT easier. Can explore QFTs using susy as a tool. Get better general understanding of QFT. Can obtain some very nontrivial exact results.

☆ Connections to math: morse thy, topological QFT, Witten awarded Fields medal for using susy to get new math.

Supersymmetric Quantum Mechanics

QM = QFT in 0+1 dimensions: $X(t)$ like $\phi(t)$ of QFT.

Susy QM algebra: $\{Q^\dagger, Q\} = 2H$, $\{Q, Q\} = 0$.

* Verify that these imply $[Q, H] = 0$.

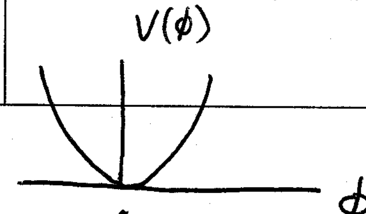
In any state $|\Omega\rangle$, $\langle\Omega|\{Q^\dagger, Q\}|\Omega\rangle$

$$= \| Q|\Omega\rangle \|^2 + \| Q^\dagger |\Omega\rangle \|^2 \geq 0$$

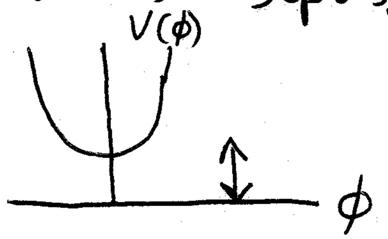
$$\therefore \langle\Omega| H |\Omega\rangle \geq 0 \implies H \geq 0$$

So potential $V(x) \geq 0$

$H = 0$ if and only if $Q|\Omega\rangle = 0$ and $Q^\dagger|\Omega\rangle = 0$. This is the condition for the state to preserve the symmetry. If there is no $E=0$ then state then there is no state with unbroken supersymmetry. The theory with no $E=0$ state "spontaneously breaks supersymmetry. E.g.



↑ could be susy vacuum here



← no $V=0$ state, breaks susy.

Q: bosons ⟷ fermions.

Consider $(-1)^F = \pm 1$, under which Q & $Q^\dagger$ have charge -1 & H has charge $+1$,

i.e. $H(-1)^F = (-1)^F H$, $\quad Q(-1)^F = -(-1)^F Q$

Consider an energy eigenstate with $(-1)^F = 1$: $|E_n, +\rangle$. Then if $E_n \neq 0$ we must have $Q|E_n, +\rangle \neq 0$ or $Q^\dagger |E_n, +\rangle \neq 0$

or both. Suppose e.g. we write

$$a = \frac{1}{\sqrt{2E_n}} Q \qquad a^\dagger = \frac{1}{\sqrt{2E_n}} Q^\dagger$$

$$\{a^\dagger, a\} = 1 \qquad \{a, a\} = 0$$ 2d Clifford algebra.

Single irrep is 2d $|+\rangle, |-\rangle$ with

$a|-\rangle = 0 \qquad a|+\rangle = |-\rangle$

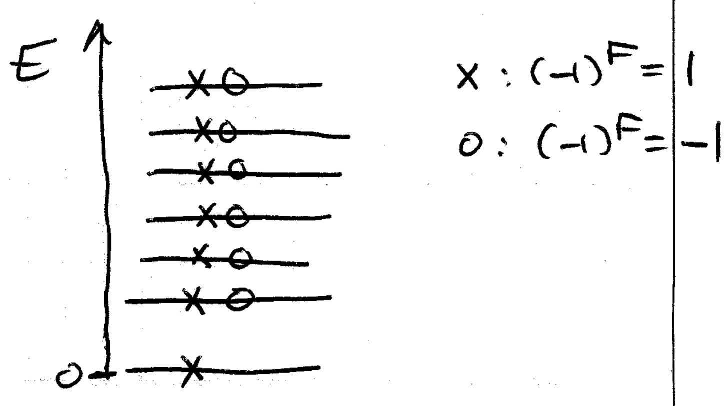
$a^+|+\rangle = 0 \qquad a^+|-\rangle = |+\rangle$

So basic irrep is e.g. $|E_n,+\rangle$
$|E_n,-\rangle$

~~saman~~. With $|E_n,+\rangle = \frac{1}{\sqrt{2E_n}} Q^+ |E_n,-\rangle$

$$|E_n,-\rangle = \frac{1}{\sqrt{2E_n}} Q |E_n,+\rangle$$

We see the bose, fermi pairs with degenerate energy for all E_n.
For $E_n = 0$ could have eg $|E_n=0,+\rangle$, which is annihilated by Q & Q^+ unpaired state at $E_n = 0$.



Witten Index:

Consider $Tr_{\mathcal{H}} (-1)^F e^{-\beta H}$. Only $H=0$ states contribute $\longrightarrow$ β indep. (Can be subtleties if spectrum is continuous rather than discrete)

$Tr_{\mathcal{H}} (-1)^F e^{-\beta H} = n_B - n_F$. n_B, n_F = # of bose & fermi states with $E_n = 0$.

$n_B + n_F$ = total # of susy ground states.

Now vary parameters of theory. Energy states can move around but all $E_n \neq 0$ states are paired, cancelling contributions to $Tr(-1)^F$. $Tr(-1)^F$ ∴ invariant under many deformations of theory. If $Tr(-1)^F \neq 0$, susy is unbroken.

Applies not only to susy QM but to general susy QFT. Will be very useful

This is all due to Witten. "Witten index".