Another example which will be useful later $\beta\gamma$

$$S = \frac{1}{2\pi}\int d^2z\, \beta\bar{\partial}\gamma$$ β, γ commuting with

β primary $(h,\bar{h}) = (\lambda, 0)$ γ primary $(h,\bar{h}) = (1-\lambda, 0)$

$$\beta(z)\gamma(w) \sim \frac{-1}{z-w} \qquad \gamma(z)\beta(w) \sim \frac{1}{z-w}$$

$$T = :(\partial\beta)\gamma: - \lambda\,\partial(\beta\gamma)$$ (* show this)

$$c = 3(2\lambda-1)^2 - 1$$

$\lambda = 3/2$ case will arise as Faddeev Popov ghosts for superstring.

Return to considering Polyakov action

$$S_P = \frac{1}{4\pi\alpha'}\int d^2\sigma\, \sqrt{-\det h}\; h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, g_{\mu\nu}$$

Take $g_{\mu\nu}$ = spacetime metric to be flat

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$ for $\mu = 0 \cdots D-1$

Take worldsheet metric $h_{ab} = e^{\phi}\eta_{ab}$

And Weyl scale ϕ away (no prob if $c_T = 0$)

$$\Rightarrow \quad S = \frac{1}{2\pi\alpha'} \int d^2z \; \eta_{\mu\nu} \, \partial X^\mu \bar{\partial} X^\nu$$

D copies of free scalar field in 2d.

Spacetime vector index $\mu = 0 \ldots D-1$ = 2d "flavor" index. Relate to our prev. normalization of ϕ: $\sqrt{\frac{2}{\alpha'}}\, X^\mu = \phi^\mu$. (Often take $\alpha' = 2$. for closed strings)

$$\langle X^\mu(z,\bar{z}) \, X^\nu(w,\bar{w}) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} \ln |z-w|^2$$

Spacetime coords X^μ = operators on 2d worldsheet

Point particle limit: $\alpha' \to 0$ & $\langle XX \rangle \to 0$.

String thy has nonlocality from $\langle XX \rangle \neq 0$ on length scales $L \leq \sqrt{\alpha'}$.

Stress tensor
$$T = -\frac{1}{\alpha'} \eta_{\mu\nu} \, \partial X^\mu \partial X^\nu$$
$$\bar{T} = -\frac{1}{\alpha'} \eta_{\mu\nu} \, \bar{\partial} X^\mu \bar{\partial} X^\nu$$

$\Rightarrow \quad c_X = D = \#$ of spacetime dimensions.

For open strings: only 1 set of oscillators & $\alpha' = 1/2$ since taking $\bar{\alpha}_n^\mu = \alpha_n^\mu$ in closed string exps.

As before: $X^\mu = \hat{x}^\mu - \frac{i\alpha'}{2}\hat{p}^\mu \ln|z|^2 + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{1}{m}\left(\frac{\alpha_m^\mu}{z^m} + \frac{\bar{\alpha}_m^\mu}{\bar{z}^m}\right)$

$[\alpha_m^\mu, \alpha_n^\nu] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}$.

Conserved currents $J_z^\mu = \sqrt{\frac{2}{\alpha'}}\, i\partial_z X^\mu$

↳ Conserved charges $\hat{p}^\mu$

Global current/charge on worldsheet = spacetime momentum = gauge charge in spacetime. General property: worldsheet global symmetries ⟶ spacetime gauge symmetries.

$\hat{p}^\mu$ eigenstates: $e^{ip^\mu X_\mu}$ = primary ops with $(h,\bar{h}) = \left(\frac{p^\mu p_\mu}{2}, \frac{\bar{p}^\mu \bar{p}_\mu}{2}\right)$. Here $p^\mu = \bar{p}^\mu$ but later generalize when some X^μ are compactified e.g. circles.

General state $\prod_i \alpha^{\mu_i}_{-n_i} |p_\mu, \bar{p}_\mu\rangle$

has $\hat{p}_\mu = p_\mu$ & $L_0 = \frac{1}{2} p^\mu p^\nu \eta_{\mu\nu} + \sum_i n_i$

Which are primary & which are descendents?

Use $T(z)\,(i\partial X^\mu(w)) = \dfrac{i\partial X^\mu}{(z-w)^2} + \dfrac{\partial(i\partial X^\mu)}{(z-w)}$

$$\Longrightarrow \quad [L_n, \alpha^\mu_m] = -m\,\alpha^\mu_{n+m}$$

e.g. $\alpha^\mu_{-1}|p\rangle$ for $|p\rangle$ primary

has $L_{n>1}(\alpha^\mu_{-1}|p\rangle) = \alpha^\mu_{n-1>0}|p\rangle = 0$

only need to check $L_1(\alpha^\mu_{-1}|p\rangle)$

$= \alpha^\mu_0 |p\rangle = p^\mu |p\rangle$ generally not primary.

Also using $L_m = \frac{1}{2}\sum :\alpha^\mu_{m-n}\alpha^\nu_n: \eta_{\mu\nu}$

$(\alpha^\mu_0 \equiv \hat{p}^\mu)$

$L_{-1}|p\rangle = \eta_{\mu\nu} p^\mu \alpha^\nu_{-1}|p\rangle$

So $\varepsilon_\mu \alpha_{-1}^\mu |p\rangle$ is only primary if $\varepsilon_\mu p^\mu = 0$ and is a descendent if ε_μ is parallel to p_μ.

For $p^\mu p_\mu = 0$ state can be both primary & descendent $\rightarrow$ "null state."

Such states have zero norm eg

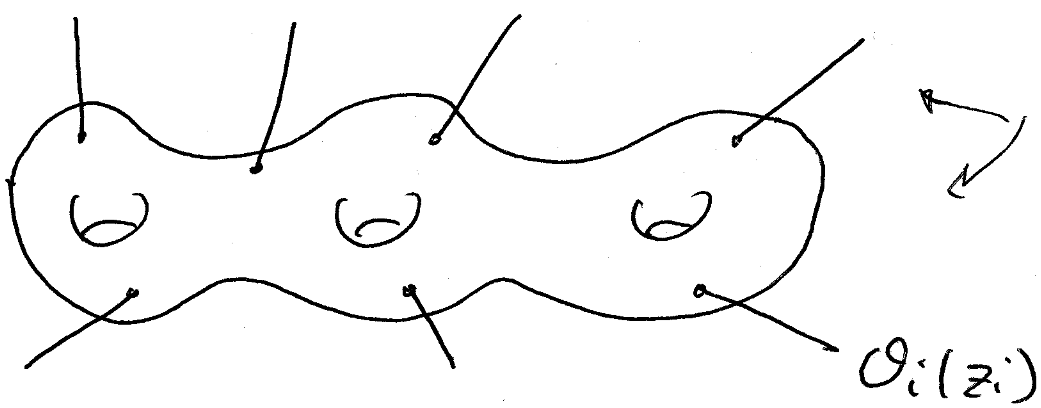
$$\| p_\mu \alpha_{-1}^\mu |p\rangle \| = p_\mu p_\nu \langle p | \alpha_1^\nu \alpha_{-1}^\mu | p \rangle$$

$$= p_\mu p_\nu \eta^{\mu\nu} \, \| |p\rangle \| = 0$$

since $p_\mu p_\nu \eta^{\mu\nu} = 0$ by assumption.

Recall generally $\langle \chi | \phi \rangle = 0$ for $|\phi\rangle$ primary & $\langle \chi |$ descendent. For $|\phi\rangle$ both primary & descendent $\Rightarrow$ $\langle \phi | \phi \rangle = 0$.

In string theory, scattering amplitudes of physical string states are represented by correlation functions of corresponding operators on 2d worldsheet. Physical states in spacetime $\longrightarrow$ operators on 2d worldsheet



L loop spacetime amplitude $\longrightarrow$ 2d worldsheet has L handles (eg above is 3 loop).
For now consider tree level, i.e. $L=0$
i.e. 2d worldsheet = sphere = our z plane with point at $z=\infty$ included. Correlation functions of operators $\langle \prod_{i=1}^{n} \mathcal{O}_i(z_i) \rangle$ is the basic ingredient in constructing spacetime amplitudes. But this, as written, would depend on operator insertion points z_i, this

is no good since the z_i coords are not really physical, we must have ~~repar~~ reparameterization inv $(\sigma, \tau) \to (\sigma'(\sigma,\tau), \tau'(\sigma,\tau))$ for arbitrary changes of our parameterization of 2d worldsheet, ie. arbitrary $(z, \bar{z}) \to (f(z,\bar{z}), \bar{f}(z,\bar{z}))$. So spacetime scattering amplitudes must be indep. of the $(z_i, \bar{z}_i)$ operator insertion points. Sol'n: integrate over the $(z_i, \bar{z}_i)$

$$\left\langle \prod_{i=1}^{n} \int d^2 z_i \, \mathcal{O}_i(z_i, \bar{z}_i) \right\rangle$$

This has an infinity which we'll identify as the group volume of the Mobius transformations and we'll have to gauge fix this to get a finite result. Deal with that shortly. For now note that we are not yet done: $\int d^2 z \, \mathcal{O}_i(\bar{z}_i, \bar{z}_i)$ is only inv. under general conformal transformations if

The operator $\mathcal{O}_i(z,\bar{z})$ is a primary operator with $(h,\bar{h}) = (1,1)$. I.e. "marginal" in 2d: $\int d^2z\, \mathcal{O}_i(z,\bar{z})$

can be added to the action without spoiling 2d scale invariance (since d^2z has $(h,\bar{h}) = (-1,-1)$). Can deform worldsheet action with sources for physical fields.

<u>Upshot</u>: The physical spacetime states correspond to primary operators with $(h,\bar{h}) = (1,1)$ on the worldsheet.

"Vertex ops"

Null states = primary & descendent give spacetime gauge invariances.

Aside: $\int d^2z$ will correspond, in point particle $\alpha' \to 0$ limit, to integral over the Schwinger parameters in a Feynman diagram. For higher loops, will also have over loop size & shape.

Consider e.g. the operator $e^{ip^\mu X_\mu}$

$\rightarrow$ state $|p_\mu\rangle$. It is primary

with $h = \bar{h} = p^2/2$. So physical

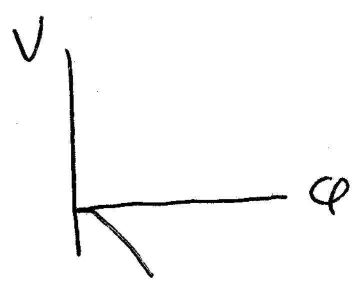
if $h = \bar{h} = 1 \Rightarrow p^2 = -m^2 = 2$.

Put back in α': $m^2 = -4/\alpha'$.

This is the "tachyon" of bosonic

string theory $\Rightarrow$ Vacuum instability

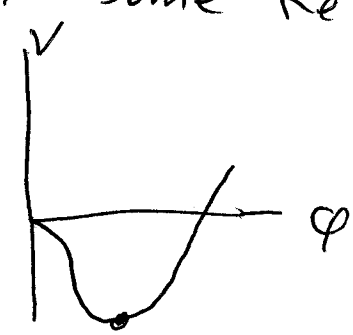
e.g. Potential



It could have a minimum for some field

expectation values $\langle\varphi\rangle$.

So not necessarily fatal.



Anyway, ~~these~~ such

tachyons are not present in superstrings.

More general states with oscillators:

- $$\varepsilon_{\mu_1 \cdots \mu_r, \bar{\mu}_1 \cdots \bar{\mu}_{\bar{r}}} \prod_{i=1}^{r} \alpha_{-n_i}^{\mu_i} \prod_{\bar{i}=1}^{\bar{r}} \bar{\alpha}_{-\bar{n}_{\bar{i}}}^{\bar{\mu}_{\bar{i}}} |p\rangle$$

polarization tensor

spin $0 \cdots r$

Has $h = N + \frac{p^2}{2}$, $\bar{h} = \bar{N} + \bar{p}^2/2$

$$N \equiv \sum_{i=1}^{r} n_i \qquad \bar{N} = \sum_{\bar{i}=1}^{\bar{r}} \bar{n}_{\bar{i}}$$

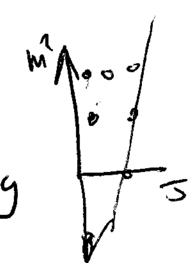
- left & right "oscillator numbers"

$p = \bar{p}$ (will later generalize in compactifications)

$h = \bar{h} = 1 \Rightarrow N = \bar{N}$ and

$$m^2 = -p^2 = 2(N-1) = \frac{4}{\alpha'}(N-1).$$

or $\frac{1}{\alpha'}(N-1)$ for open strings



- Tower of massive states, separated by gap $4/\alpha' \sim M_{\text{planck}}^2$. $p(N)$ type degeneracy. (spin $0 \cdots N$.)

But not all of these states are primary. Must verify that all $L_{n>0}$, $\bar{L}_{n>0}$ annihilate the state. Also freedom to shift by null states (annihilated by $L_{n>0}, \bar{L}_{n>0}$ but still descendents of form ~~[illegible]~~ $|n\rangle = L_{-k}|\chi\rangle$), which is gauge equivalence. These conditions put constraints on the allowed polarization tensors. Use $[L_n, \alpha^\mu_m] = -m\alpha^\mu_{n+m}$ to pull $L_{n>0}$ through oscillators

$$\prod_{i=1}^{\hat{r}} \alpha^{\mu_i}_{-n_i} \quad \prod_{\bar{i}=1}^{\bar{r}} \bar{\alpha}^{\bar{\mu}_i}_{-\bar{n}_i}.$$

Will get zero if any $L_{n>0}$ or $\alpha^\mu_{n>0}$ acts on $|p\rangle$ at end. Will find that in critical dimension $D=26$, $D-2$ independent ~~[illegible]~~ light cone oscillators $\alpha^\mu_{-n_i}$ values of μ left after imposing primary cond. & gauge inv. equiv.

Open strings: Must specify B.C. on ends of string. Want 2d worldsheet stress tensor T_{ab} to be conserved. Consider a wave pulse on string  we want the power carried to be entirely reflected back upon hitting the end. Two choices: nail down end or have it free. So at boundary $\sigma = 0$ or $\sigma = \pi$ can have either:

① $X = X_0$ fixed at boundary (no displacement)

so $\frac{\partial}{\partial \tau} X \Big|_{\sigma \text{ on bndy}} = 0$. This is

"Dirichlet" boundary condition. ← (nailed down)

② free B.C. $\Rightarrow$ $\frac{\partial}{\partial \sigma} X \Big|_{\sigma \text{ on bndy}} = 0$

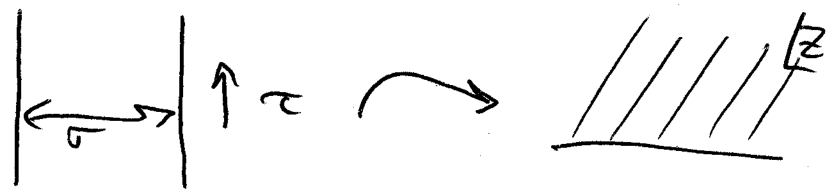
this is "Neumann" boundary condition. ← (free)

Write as Dirichlet D: $t^a \partial_a X = 0$
Neumann N: $n^a \partial_a X = 0$

on boundary $\partial\Sigma$ of 2d worldsheet, $t^a =$ tangent to $\partial\Sigma$ and $n^a =$ normal to $\partial\Sigma$

Nailing down a end breaks the spacetime translation invariance, which leads to an object known as a D brane. More on this later. For now just consider the case of free or Neumann ends.

Mapping strip



to upper ½ of z plane. The boundary is the line $z = \bar{z}$ and the B.C. is

$$\partial X(z,\bar{z}) = \bar{\partial} X(z,\bar{z}) \quad \text{at} \quad z = \bar{z}.$$

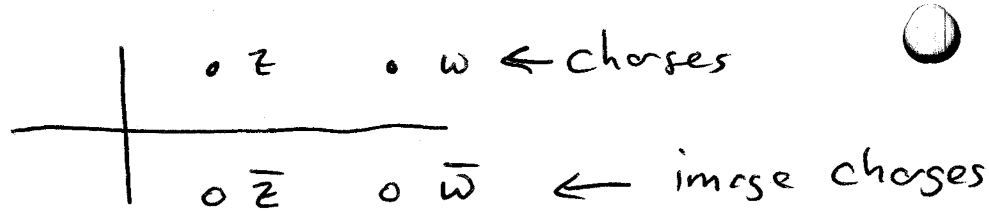
(writing $z = x + iy$, this is $\frac{\partial}{\partial y} X(x, y=0) = 0$.)

Take these BCs for all $\mu = 0 \dots D$.

Our closed string two point function

$$\langle X^\mu(z,\bar{z})\, X^\nu(w,\bar{w})\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu} \ln|z-w|^2$$

doesn't satisfy this B.C. need image charges



For open strings (with all N BCs)

$$\langle X^\mu(z,\bar z)\, X^\nu(w,\bar w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\left[\ln|z-w|^2 + \ln|z-\bar w|^2\right]$$

$$[\,] = \ln(z-w) + \ln(\bar z - \bar w) + \ln(z-\bar w) + \ln(\bar z - w)$$

↑ Greens fn between each charges/ image charges.

Satisfies $(\partial_z - \partial_{\bar z})[\,]\Big|_{z=\bar z} = 0$ for all w.

On boundary $z=\bar z$, $w=\bar w$

4 terms in [ ]

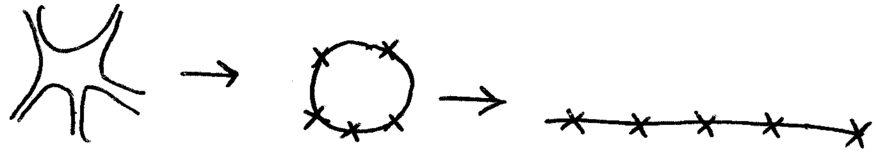
$$\langle X^\mu(z)\, X^\nu(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}(4)\ln(z-w)$$

$$= -2\alpha'\eta^{\mu\nu}\ln(z-w)$$

∴ Similar to previous free boson formulae if we set $\alpha' = 1/2$, v.s. closed string where its similar if we set $\alpha'=2$.

E.g. $e^{ipX(z,\bar z)}$ on boundary $z=\bar z$

has $h = (2\alpha')\frac{p^2}{2}$ (vs $\left(\frac{\alpha'}{2}\right)\left(\frac{p^2}{2}\right)$ for operator in bulk.)

Open strings scattering 

Physical open string states $\longrightarrow$ operators on boundary with $\underline{h=1}$ ~~[illegible]~~ & $\underline{\text{primary}}$ to have $\oint_{\partial\Sigma} dz\, V(z)$ be reparameterization invt under $z \to f(z)$. E.g. boundary op $V(z) = e^{ipX(z)}$ is primary & has $h = (2\alpha')\left(\frac{p^2}{2}\right) = 1$ for $p^2 = -m^2 = 1/\alpha'$.

"Boundary tachyon"

vs. closed string tachyon $e^{ipX(z,\bar{z})}$ in bulk of Worldsheet with $m^2 = -4/\alpha'$.

More general open string boundary states:

$$\varepsilon_{\mu_1 \ldots \mu_r} \prod_{i=1}^{r} \alpha_{-n_i}^{\mu_i} |p\rangle \qquad \text{spin } 0 \ldots r$$

has $h = N + \alpha' p^2 \qquad N = \sum_{i=1}^{r} n_i$

$h=1$ if $m^2 = -p^2 = \frac{1}{\alpha'}(N-1)$. Tower of massive states w/ $\frac{1}{\alpha'}$ gap. Again not all primary.

Look for sol'ns of physical state cond for $N = 1$ oscillator (open) & $N = \bar{N} = 1$ (closed)

Operators: $\varepsilon_\mu \partial X^\mu e^{ipX}$ (open)

$\varepsilon_{\mu\bar{\nu}} \partial X^\mu \bar{\partial} X^{\bar{\nu}} e^{ipX}$ (closed)

→ states

$\varepsilon_\mu \alpha_{-1}^\mu |p\rangle$ (open)

$\varepsilon_{\mu\bar{\nu}} \alpha_{-1}^\mu \bar{\alpha}_{-1}^{\bar{\nu}} |p\rangle$ (closed)

Open case $h = 1 \leftrightarrow p^2 = 0$ massless
Closed $h = \bar{h} = 1 \leftrightarrow p^2 = 0$.

Now impose primary operator / state cond:
State must be annihilated by all $L_{n>0}$

Use $[L_n, \alpha_m^\mu] = -m\, \alpha_{n+m}^\mu$

e.g. $L_{n>0}\, \alpha_{-1}^\mu |p\rangle = [L_{n>0}, \alpha_{-1}^\mu] |p\rangle$

(since $L_{n>0} |p\rangle = 0$)

$= \alpha_{n-1}^\mu |p\rangle = p^\mu \delta_{n,1} |p\rangle$

$\therefore$ $\varepsilon_\mu \alpha_{-1}^\mu |p\rangle$ is primary $\leftrightarrow \varepsilon_\mu p^\mu = 0$.

Note that this eliminates e.g. $\varepsilon_\mu = \delta_{\mu,0}$ which would have negative norm.

Similarly, for the closed string, $\varepsilon_{\mu\bar{\nu}}\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\bar{\nu}}|p\rangle$ is annihilated by all $L_{n>0}$ & $\bar{L}_{n>0}$

$$\longleftrightarrow \quad \varepsilon_{\mu\bar{\nu}}\, p^{\mu} = \varepsilon_{\mu\bar{\nu}}\, p^{\bar{\nu}} = 0.$$

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* Consider the <u>open</u> string state $\varepsilon_{\mu\nu}\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}|p\rangle$ and find the conditions on $\varepsilon_{\mu\nu}$ & p_{μ} for it to be primary with $h=1$.

We can shift the open string state

$$\varepsilon_{\mu}\alpha_{-1}^{\mu}|p\rangle \quad \text{by} \quad \lambda L_{-1}|p\rangle$$

for any λ since this state is <u>null</u> for $p^2=0$: $\quad L_{n>0}L_{-1}|p\rangle =$

$$[L_{n>0}, L_{-1}]|p\rangle = (n+1)L_{n-1}|p\rangle = 0$$

since $L_{m>0}|p\rangle = 0$ and $L_0|p\rangle = 2\alpha'\frac{p^2}{2} = 0$ for $p^2=0$. Also $L_{-1}|p\rangle$ has L_0 eigenvalue $h=1$ for $p^2=0$.

Using $L_m = \frac{1}{2}\sum_n :\alpha^{\mu}_{m-n}\alpha^{\nu}_{n}:\eta_{\mu\nu}$

and $\alpha^{\mu}_{m\geq 0}|p\rangle = p^{\mu}\delta_{m,0}|p\rangle$

Find $L_{-1}|p\rangle = \eta_{\mu\nu}\alpha^{\mu}_{-1}\alpha^{\nu}_{0}|p\rangle$

$= p_\mu \alpha^{\mu}_{-1}|p\rangle,$

So we can freely shift $\varepsilon_\mu \to \varepsilon_\mu + \lambda p_\mu$ in state ~~...~~ $\varepsilon_\mu \alpha^{\mu}_{-1}|p\rangle.$

The state $\varepsilon_\mu \alpha^{\mu}_{-1}|p\rangle$ is the massless photon, $p^\mu \varepsilon_\mu = 0$ is Gauss' law, and $\varepsilon_\mu \to \varepsilon_\mu + \lambda p_\mu$ is gauge inv.

So get physical photon with $D-2$ physical polarization compts of ε_μ. Correct Lorentz repn ~~with ...~~ Illustrates general prop: Spacetime gauge inv. $\longleftrightarrow$ Shifts by null states.

Similarly for closed string, can freely shift $\varepsilon_{\mu\bar\nu}\alpha^{\mu}_{-1}\bar\alpha^{\bar\nu}_{-1}|p\rangle$ by

$a_\mu \bar L_{-1}\alpha^{\mu}_{-1}|p\rangle + b_{\bar\nu} L_{-1}\bar\alpha^{\bar\nu}_{-1}|p\rangle$

which is descendent but also primary with $h=\bar h=1$ for any a_μ, $b_{\bar\nu}$ and $p^2=0$

$\Rightarrow$ Can freely shift $\varepsilon_{\mu\bar{\nu}} \to \varepsilon_{\mu\bar{\nu}} + a_\mu p_{\bar{\nu}} + b_{\bar{\nu}} p_\mu$

again, this corresponds to spacetime gauge symmetry. Write $\varepsilon_{\mu\nu}$ as 3 parts

① traceless & symmetric $\varepsilon_{\mu\nu} = S_{\mu\nu}$

$\longrightarrow$ spin 2 in spacetime = graviton!

Above gauge inv = usual reparam. gauge inv.

② antisymmetric $\varepsilon_{\mu\nu} = a_{\mu\nu}$

$\to$ $B_{\mu\nu}$ gauge field with $B_{\mu\nu} = -B_{\nu\mu}$

"two form gauge field", with gauge inv

$B_{\mu\nu} \to B_{\mu\nu} + a_\mu p_\nu - a_\nu p_\mu$ i.e. $\underset{\text{2 form}}{B} \to B + d\underset{\text{1 form}}{\Lambda}$

Just as A_μ couples to worldline of pt particles via $\int A_\mu dx^\mu$, $B_{\mu\nu}$ couples to string worldvolume via $\int B_{\mu\nu} dx^\mu \wedge dx^\nu$ (more soon). B's field strength is $H = dB$, i.e. $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$.

③ $\varepsilon_{\mu\nu}$ = trace part = $\eta_{\mu\nu}$ (+ p^μ terms for $\varepsilon \cdot p = 0$)

$\eta_{\mu\nu} - p_\mu \bar{p}_\nu - \bar{p}_\mu p_\nu$
with $\bar{p} \cdot \bar{p} = 0$ & $p \cdot \bar{p} = 0$

This corresponds to the dilaton scalar Φ.

The physical state cond: primary with $h = \bar{h} = 1$, Would lead to negative norm physical states if $c_X = D > 26$

eg $\left(\alpha_{-1} \cdot \alpha_{-1} + \frac{D-1}{5} p \cdot \alpha_{-2} + \frac{(D+4)}{10} (p \cdot \alpha_{-1})^2 \right) |p\rangle$

with $+2 + \frac{p^2}{2} = 1$. Annihilated by all $L_{n>0}$ & $h = 1$ (& need similar $\bar{\alpha}$ ops for $\bar{h} = 1$, don't bother writing here)

& This state has $\langle \phi | \phi \rangle = \frac{2}{25} (D-1)(26-D) \langle p | p \rangle$

$\Rightarrow$ Better not have $D > 26$! (unless this state is somehow eliminated)

"No ghost thm" for $D \leq 26$

no neg norm physical sts if $D \leq 26$

For $D = 26$ many null states eg.

$|\psi\rangle = (L_{-2} + \frac{3}{2} L_{-1}^2) |\chi\rangle$ is primary & has $h = 1$ if $|\chi\rangle$ is primary with $h = -1$. $|\psi\rangle$=Primary & descendant

$\therefore$ $\langle \psi | \psi \rangle = 0$. Null states $\rightarrow$ extra gauge symm.

For $C_X = D \leq 26$ Find physical states condition eliminates 1 dimension worth of oscillators $\sim D-1$ indep oscillators.

For $C_X = D = 26$, Extra gauge symm of add'l null states eliminates another dim worth of oscillators $\longrightarrow D-2=24$ eff. indep. oscillators. Expected $D-2$ "light cone" dimensions since τ & σ worldsheet reparam inv can be used to eliminate 2 polarization directions.

Will give better explanation soon via ghosts.

$$C_X + C_{ghosts} + C_{Liouville} = 0$$

$$C_{ghosts} = -26 \qquad C_X = D$$

Can neglect $C_{Liouville}$ if $D = 26$. Otherwise it is the dim. not eliminated by extra gauge symms.