Compactification of Spacetime

- Consider a worldsheet scalar ~ spacetime coordinate X with periodic identification

$$X \simeq X + 2\pi R \qquad S_{ws} = \int \frac{d^2z}{2\pi\alpha'} \partial X \bar{\partial} X$$

Vertex ops e^{ipX}, now $p = n/R$ quantized to be invt under $X \simeq X + 2\pi R$. As in pt. particle phys with $n =$ integer.

- But can also have $X(\sigma + 2\pi, \tau) = X(\sigma, \tau) + 2\pi R w$ for arbitrary "winding number" integer w. Strings can wind around circle in spacetime.

$$X = X_L(z) + X_R(\bar{z}) \qquad \begin{aligned} \langle X_L(z) X_L(w) \rangle &= -\tfrac{\alpha'}{2} \ln(z-w) \\ \langle X_R(\bar{z}) X_R(\bar{w}) \rangle &= -\tfrac{\alpha'}{2} \ln(\bar{z}-\bar{w}) \end{aligned}$$

$$X_L = \hat{X}_L - i\hat{p}_L \frac{\alpha'}{2} \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\alpha_m}{m z^m}$$

$$X_R = \hat{X}_R - i\hat{p}_R \frac{\alpha'}{2} \ln \bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\bar{\alpha}_m}{m \bar{z}^m}$$

- Under $\sigma \to \sigma + 2\pi \qquad z \to e^{2\pi i} z$

$$X = X_L + X_R \longrightarrow X + \frac{2\pi\alpha'}{2}\left(\hat{p}_L - \hat{p}_R\right) \overset{!}{=} X + 2\pi R w$$

So $\hat{p}_L - \hat{p}_R = \dfrac{2Rw}{\alpha'}$ & $\hat{p} = \dfrac{\hat{p}_L + \hat{p}_R}{2} = \dfrac{n}{R}$

So
$$\left.\begin{aligned} p_L &= \frac{n}{R} + \frac{wR}{\alpha'} \\ p_R &= \frac{n}{R} - \frac{wR}{\alpha'} \end{aligned}\right\}$$
eigenvalues of separate left & right conserved charges $\hat{p}_L$ & $\hat{p}_R$

The conserved charges $\hat{p}_L$ & $\hat{p}_R$ are associated with the conserved currents

$$J_L(z) = \frac{i2}{\alpha'}\partial X \qquad J_R(\bar{z}) = \frac{i2}{\alpha'}\bar{\partial} X$$

$\left(\frac{2i}{\alpha'}\right) \rightarrow$

which are separately conserved: $\bar{\partial} J_L = 0$, $\partial J_R = 0$

(thanks to $\partial\bar{\partial}X = 0$ EOM)

J_L has $(h,\bar{h}) = (1,0)$ & J_R has $(h,\bar{h}) = (0,1)$

$\hat{p}_L$ & $\hat{p}_R$ eigenstates (operators): $\mathcal{O}_{p_L, p_R} = e^{i(p_L X_L + p_R X_R)}$

with above p_L, p_R eigenvalues

So for any R there is a global $U(1)_L \times U(1)_R$ symmetry on the worldsheet.

A general result in string theory:

$$\left.\begin{array}{l}\text{Global worldsheet}\\ \text{symmetries}\end{array}\right] \longrightarrow \left[\begin{array}{l}\text{Gauge spacetime}\\ \text{symmetries}\end{array}\right.$$

always true. Reason: consider a global worldsheet current J_L with $(h,\bar{h}) = (1,0)$ or J_R with $(h,\bar{h}) = (0,1)$. Could e.g. be ones we were just looking at, but let's keep it general. Then can always form $(h,\bar{h}) = (1,1)$ vertex operators as

$$\left\{\begin{array}{l}\varepsilon_{\mu}\,\bar{\partial}X^{\mu}\,J_L(z)\,e^{ik\cdot X}\\ \varepsilon_{\mu}\,\partial X^{\mu}\,J_R(\bar{z})\,e^{ik\cdot X}\end{array}\right.$$

both are (1,1) primary if $k^2 = 0$ & $k\cdot\varepsilon = 0$. Mod null states $L_{-1}|\bar{\chi}\rangle$ or $\bar{L}_{-1}|\chi\rangle$ with $|\bar{\chi}\rangle$ & $|\chi\rangle$ primary with $(h,\bar{h}) = (0,1)$ or $(1,0)$

$\Longrightarrow$ mod $\varepsilon_{\mu} \to \varepsilon_{\mu} + \lambda k_{\mu}$. So these vertex operators are making usual gauge fields (in momentum space)

Just like our previous open string gauge field vertex operators, but here in closed string. The currents J_L or J_R contribute the extra $(h,\bar{h}) = (1,0)$ or $(0,1)$ needed to turn these gauge field vertex operators into $(h,\bar{h}) = (1,1)$ closed string vertex operator. So any current J_L or J_R always leads to spacetime gauge symmetry. The spacetime gauge charge = the worldsheet global charge.

$\therefore$ For $X \simeq X + 2\pi R \longrightarrow U(1)_L \times U(1)_R$ gauge fields living in uncompact directions from $U(1)_L \times U(1)_R$ global worldsheet symmetries. The charge of $U(1)_L$ is p_L & that of $U(1)_R$ is p_R.

A general vertex operator has

$$L_0 = \frac{\alpha'}{4} p^2 + \frac{\alpha'}{4} p_L^2 + N$$

$$\bar{L}_0 = \frac{\alpha'}{4} p^2 + \frac{\alpha'}{4} p_R^2 + \bar{N}$$

total oscillator # (including compactified & non compact dirs)

p = momentum in uncompactified directions

Setting $L_0 = \bar{L}_0 = 1$ for physical states & $p^2 = -m^2$

get $m^2 = p_L^2 + \frac{4}{\alpha'}(N-1) = p_R^2 + \frac{4}{\alpha'}(\bar{N}-1)$

Our massless $U(1)_L \times U(1)_R$ gauge fields have

$p_L = p_R = 0$ (good, $U(1)$ gauge fields should

be neutral) & $N = \bar{N} = 1$:

$U(1)_L$ gauge field: $\varepsilon_{\bar{\mu}} \bar{\partial} X^{\bar{\mu}} J_L e^{ip.X} \sim \varepsilon_{\bar{\mu}} \bar{\partial} X^{\bar{\mu}} \partial X^{25} e^{ip.X}$

$U(1)_R$ gauge field: $\varepsilon_{\mu} \partial X^{\mu} J_R e^{ip.X} \sim \varepsilon_{\mu} \partial X^{\mu} \bar{\partial} X^{25} e^{ip.X}$

Where we're calling the compact direction X^{25}.

The above gauge field vertex ops are our

previous closed string vertex ops: $\varepsilon_{\mu\bar{\nu}} \partial X^{\mu} \bar{\partial} X^{\bar{\nu}} e^{ip.X}$

With μ or $\bar{\nu}$ = the compact dir X^{25}.

Recall $\varepsilon_{\mu\bar{\nu}} = S_{\mu\bar{\nu}} + a_{\mu\bar{\nu}}$ symm &

antisymm, with the symmetric traceless

part = graviton & antisymmetric part = B field

So $U(1)_L$ gauge field $\sim \varepsilon_{\mu 25}\,\bar{\partial} X^{\mu}\,\partial X^{25}\, e^{ipX}$

$U(1)_R$ gauge field $\sim \varepsilon_{\mu 25}\,\partial X^{\mu}\,\bar{\partial} X^{25}\, e^{ipX}$

symmetric part : $U(1)_{\frac{L+R}{2}} = \frac{1}{2}(A_{\mu L} + A_{\mu R}) = g_{\mu 25}$

antisymmetric part $U(1)_{\frac{L-R}{2}} = \frac{1}{2}(A_{\mu L} - A_{\mu R}) = B_{\mu 25}$

The $U(1)_{\frac{L+R}{2}}$ charge $= \frac{1}{2}(p_L + p_R) = \frac{n}{R}$

$U(1)_{\frac{L-R}{2}}$ charge $= \frac{1}{2}(p_L - p_R) = \frac{wR}{\alpha'}$.

Note $U(1)_{\frac{L+R}{2}}$ is the usual Kaluza Klein $U(1)$ with gauge field $\sim g_{\mu 25}$ & charge = momentum in compact direction. $U(1)_{\frac{L-R}{2}}$ is new, it's gauge field $\sim B_{\mu 25}$ & charge = winding # in compact direction. To see that the B field couples to winding # write $\int B_{\mu 25}\, dX^{\mu} \wedge dX^{25}$ $= 2\pi w R \int B_{\mu 25}\, dX^{\mu}$, where we used $\int dX^{25} = w\, 2\pi R_0$ of form $q \int A_{\mu} dX^{\mu}$, with $A_{\mu} = B_{\mu 25}$ & $q = 2\pi R w$.

For $\alpha' \to 0$ $U(1)_{\frac{L-R}{2}}$ decouples, all charged fields have ∞ mass.

~~Note $p_L, p_R = \frac{n}{R} \pm \frac{wR}{\alpha'}$~~

From $\mathcal{O}_{p_L, p_R}$ form states $|p_L, p_R\rangle$

& more generally $\prod_{i,\bar{\imath}} \alpha_{-n_i} \bar{\alpha}_{-\bar{n}_{\bar{\imath}}} |p_L, p_R\rangle$

As before
$$L_0 = \frac{\alpha' p_L^2}{4} + \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n$$
$$\bar{L}_0 = \frac{\alpha' p_R^2}{4} + \sum_{n=1}^{\infty} \bar{\alpha}_{-n} \bar{\alpha}_n$$

Note
$$\begin{matrix} p_L \\ p_R \end{matrix} = \frac{n}{R} \pm \frac{wR}{\alpha'}$$

has a symmetry under : $R \to \alpha'/R$

with $n \leftrightarrow w$. Takes $p_L \to p_L$, $p_R \to -p_R$.

Theory with $X = X_L + X_R$ on circle of radius R has same spectrum as that with $\tilde{X} = X_L - X_R$ on circle of radius α'/R. This is actually an exact symmetry of the CFT and an exact symmetry of string theory (even non-perturbatively)! Strings can't tell the difference between a spacetime geometry with a circle of radius R & that with radius α'/R. Geometry somewhat ambiguous!

"T duality": $R \to \alpha'/R$

Something special happens at $R = \sqrt{\alpha'}$ the self-dual radius: $p_{L,R} = \left(\frac{n \pm m}{\sqrt{\alpha'}}\right)$

A state $\prod_{i,\bar{i}} \alpha_{-n_i} \bar{\alpha}_{-\bar{n}_{\bar{i}}} |p_L, p_R\rangle$

with $\sum' n_i = N$ & $\sum' \bar{n}_{\bar{i}} = \bar{N}$

has $L_0 = h = \frac{1}{4}(n+m)^2 + N$

$$\bar{L}_0 = \bar{h} = \frac{1}{4}(n-m)^2 + \bar{N}$$

For $n = m = \pm 1$ & $N = \bar{N} = 0$

get two operators with $(h, \bar{h}) = (1, 0)$:

$$J_\pm (z) = e^{\pm \frac{i2}{\sqrt{\alpha'}} X_L(z)}$$

$\leftarrow$ new left moving currents!

$\bar{\partial} J_\pm = 0$

For $n = -m = \pm 1$ & $N = \bar{N} = 0$ get

$$\bar{J}_\pm (\bar{z}) = e^{\pm \frac{i2}{\sqrt{\alpha'}} X_R(\bar{z})}$$

$\leftarrow$ new right moving currents

$\partial \bar{J}_\pm = 0$

$J_\pm$ carries charge ± 1 under $J_3 \equiv i \frac{1}{\sqrt{\alpha'}} \partial X$

The corresponding conserved charges satisfy

$[Q_i, Q_j] = i \varepsilon_{ijk} Q_k$ SU(2) commutation rel'ns

$$Q_3 \equiv \frac{\boxed{\alpha'} \hat{P}_L}{2} = \oint \frac{dz}{2\pi i} J_3 , \quad Q_\pm \equiv Q_1 \pm i Q_2 = \oint \frac{dz}{2\pi i} J_\pm .$$

Simarly for $\bar{J}_\pm$: get $U(1)_L \times U(1)_R$

enhanced to global $SU(2)_L \times SU(2)_R$ at

self-dual radius $R = \sqrt{\alpha'}$!

Since global symms on worldsheet $\longleftrightarrow$ gauge symm in spacetime, get $U(1)_L \times U(1)_R$ spacetime gauge symmetry enhanced to $SU(2)_L \times SU(2)_R$ gauge symmetry of spacetime string theory when on a self-dual radius circle.

Going from $R = \sqrt{\alpha'} \longrightarrow R \neq \sqrt{\alpha'}$ the $SU(2)_L \times SU(2)_R$ is broken to $U(1) \times U(1)$ by expectation values of massless fields in $(3,3)$. These fields are $\leftrightarrow$ vertex ops $e^{ikX} J_i(z) \bar{J}_j(\bar{z})$ where $i, \bar{j} = 3, \pm$.

These are $(1,1)$ primary for $k^2 = 0$ (massless).

Since $SU(2)_L \times SU(2)_R$ at $R = \sqrt{\alpha'}$ is a <u>gauge</u> symm it must be exact i.e. respected by all string loops & even non-perturbatively (otherwise theory would be inconsistent!). $R \leftrightarrow \alpha'/R$ is the $\mathbb{Z}_2$ Weyl subgp. left unbroken & $\therefore$ also exact, even non-perturbatively!

Partition function for X:

$$Z = (q\bar{q})^{-1/24}\, \mathrm{Tr}\left(q^{L_0}\, \bar{q}^{\tilde{L}_0}\right) = \frac{1}{\eta\bar{\eta}} \sum_{n,w=-\infty}^{\infty} q^{\frac{\alpha' p_L^2}{4}}\, \bar{q}^{\frac{\alpha' p_R^2}{4}}$$

$$= \frac{1}{|\eta(\tau)|^2} \sum_{n,w=-\infty}^{\infty} \exp\left[2\pi i \tau_1 n w - \pi\tau_2\left(\frac{\alpha' n^2}{R^2} + \frac{w^2 R^2}{\alpha'}\right)\right]$$

Use $\displaystyle\sum_{n=-\infty}^{\infty} \exp\left(2\pi i b n - \pi a n^2\right) = a^{-1/2} \sum_{m=-\infty}^{\infty} \exp\left(\frac{-\pi(m-b)^2}{a}\right)$

$$\Rightarrow Z = 2\pi R\, \underbrace{Z_X(\tau)}_{(4\pi^2\alpha'\tau^2)^{-1/2}|\eta(\tau)|^2} \sum_{m,w=-\infty}^{\infty} \exp\left(-\frac{\pi R^2 |m - w\tau|^2}{\alpha' \tau_2}\right)$$

invt under $SL(2,\mathbb{Z})$.

* Since $Z_X(\tau)$ is $SL(2,\mathbb{Z})$ invt, only need to verify that the $\displaystyle\sum_{m,w=-\infty}^{\infty} \exp\left(-\frac{\pi R^2 |m-w\tau|^2}{\alpha'\tau_2}\right)$ above

also is $SL(2,\mathbb{Z})$ invt. under $\tau \to \frac{a\tau+b}{c\tau+d}$ with $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$. Show this.

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above sum formula is "Poisson resummation" follows from $\displaystyle\sum_m e^{2\pi i m r} = \sum_n \delta(r-n)$ & Gaussian integral

$$X(\sigma^1 + 2\pi, \sigma^2) = X(\sigma^1, \sigma^2) + 2\pi w R$$

$$X(\sigma^1 + 2\pi\tau_1, \sigma^2 + 2\pi\tau_2) = X(\sigma^1, \sigma^2) + 2\pi m R$$

2 cycles of worldsheet torus wrap w & m times around spacetime circle of radius R.

$$X_{cl} = \sigma^1 w R + \sigma^2 (m - w\tau_1) R/\tau_2$$

$$\partial^a \partial_a X_{cl} = 0 \checkmark$$

$$X = X_{cl} + X_{quantum} \qquad S = S_{cl} + S_{quant}$$

$$\int [dX]\, e^{-S} = \underbrace{\int [dX_q]\, e^{-S_q}}_{2\pi R Z_X(\tau)} \sum_{m,w} e^{-S_{cl}(m,w)}$$

* verify $\quad S_{cl} \equiv \frac{1}{2\pi\alpha'} \int_{torus} d\sigma^1 d\sigma^2 \left(\partial_a X_{cl} \partial^a X_{cl}\right)$

$$= \frac{\pi R^2 |m - w\tau|^2}{\alpha' \tau_2}$$

Open Strings: Recall two choices of BCs on ends such that worldsheet stress tensor $\Rightarrow$ energy carried along string is conserved:

① $X = X_0$ fixed at boundary: $\left.\frac{\partial X}{\partial \tau}\right|_{\sigma\, \text{bndy}} = 0$

i.e. $t^a \partial_a X = 0$ $\quad t^a$ = tangent to bndy

this is "Dirichlet" BC

② free b.c. $\Rightarrow$ $\left.\frac{\partial X}{\partial \sigma}\right|_{\sigma\, \text{bndy}} = 0$

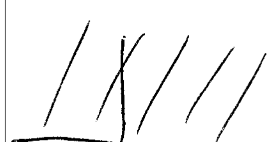
i.e. $n^a \partial_a X = 0$ $\quad n^a$ = normal to bndy

this is Neumann.

Dirichlet B.C. breaks spacetime translation inv momentum p in this direction is not conserved.

Neumann might $\therefore$ seem "better". Map strip to

z ~~plane~~ upper 1/2 plane



Neumann B.C.: $\partial X(z,\bar z) = \bar\partial X(z,\bar z)$ at $z = \bar z$

Now consider Neumann open string on circle of radius R.

$$X \sim X + 2\pi R$$

With Neumann B.C.s momentum around circle is conserved but winding number w is not, since closed wound strings can now snap into open strings, which can unwind.

Closed string sector: $X \sim X + 2\pi R$

is T dual to $\tilde{X} \sim \tilde{X} + 2\pi \tilde{R}$ $\quad \tilde{R} \equiv \alpha'/R$

with $n \leftrightarrow w$ momentum $\leftrightarrow$ winding modes

For open string sector $\tilde{X}$ now has Dirichlet rather than Neumann BCs:

$\tilde{X} \equiv X_L(z) - X_R(\bar{z})$ for $X \equiv X_L(z) + X_R(\bar{z})$

if $\partial X = \bar{\partial} X$ at $z = \bar{z}$ $\Rightarrow$ $\partial X_L = \bar{\partial} X_R$
at $z = \bar{z}$ real

$\Rightarrow \partial \tilde{X} = -\bar{\partial} \tilde{X}$ at $z = \bar{z}$

$t^a \partial_a \tilde{X} = n^a \partial_a X$ & visa versa

T duality exchanges: $D \leftrightarrow N$ BCs

$$X_L = -\frac{i\alpha'}{2} p \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{\alpha_m}{m z^m}$$

$$X_R = -\frac{i\alpha'}{2} p \ln \bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{\alpha_m}{m \bar{z}^m}$$

satisfy $\partial X_L = \bar{\partial} X_R$ at $z = \bar{z}$

$X = X_L + X_R$ $\quad p = \text{momentum} = \frac{n}{R}$

$\tilde{X} = X_L - X_R$ $\quad p = \text{Winding} = \frac{m\tilde{R}}{\alpha'}$

T duality exchanges $R \leftrightarrow \tilde{R} = \alpha'/R$ & momenta & winding. Although Dirichlet BCs break translation inv $\rightarrow$ momentum not conserved, they conserve winding number.

T duality: should also consider D B.C. objects $\rightarrow$ D branes.

Dp brane: $X^0, X^1, \cdots X^{p-1}$ N BCs

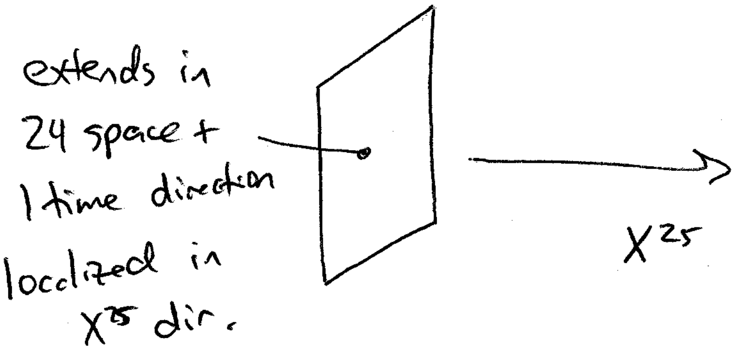
rest = D BCs.

Usual open string = "D25 brane"

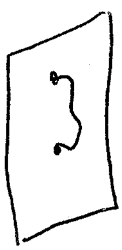
Consider D bcs in $X \equiv X^{25}$ direction &

N in rest $\longrightarrow$ D24 brane. At point

$X^{25} = X^0$ fixed, e.g. $X^0 = 0$



Open string ends must lie in D brane



No momentum in X^{25} direction, open strings are confined to live on the D brane. Physical open string states: $L_0 = \alpha' p^2 + N = 1$

where p lives in brane directions only

$\longrightarrow \quad m^2 = \frac{1}{\alpha'}(N-1)$ spectrum of states confined to brane.

$N = 0 \longrightarrow$ tachyon

$N = 1 \longrightarrow$ massless.
$$\left\{ \begin{array}{ll} \alpha_{-1}^{25} \, |p\rangle & \\ \alpha_{-1}^{\mu} \, |p\rangle & \mu = 0 \ldots 24 \end{array} \right.$$

$\alpha_{-1}^{25} |p\rangle \longrightarrow$ Scalar

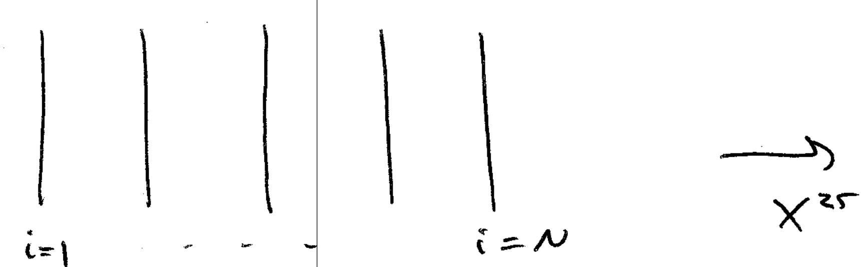
$\alpha_{-1}^{\mu} |p\rangle \longrightarrow$ U(1) gauge field living on brane

The scalar is $\sim X^{25}(X^{\mu})$ whose expectation value gives location of D brane in X^{25} direction

D branes become dynamical. Function of coords X^{μ} in brane.

Consider n D branes, at $X^{25} = V^i$

$i = 1 \ldots n$



Open strings can have one end on the i^{th} brane & other end on j^{th} brane

$$X^{25} = V^i + \underbrace{\frac{\sigma^1}{\pi}(V^j - V^i)} + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^{25}}{m}\left(\frac{1}{z^m} - \frac{1}{\bar{z}^m}\right)$$

$\frac{\sigma_1}{\pi} = \frac{-i}{2\pi}\ln\left(\frac{z}{\bar{z}}\right)$

$\hookrightarrow$ no momentum in X^{25} dir but Winding # $= \dfrac{V^j - V^i}{2\pi\alpha'}$

Leads to states in i & j branes with masses $m^2 = \left(\dfrac{V^i - V^j}{2\pi\alpha'}\right)^2 + \dfrac{1}{\alpha'}(N-1)$

Extra contribution $\left(\frac{V_i - V_j}{2\pi\alpha'}\right)$ to mass has a simple interpretation:

$$m^2 = (TL)^2 + \frac{1}{\alpha'}(N-1)$$

T = string tension $= 1/2\pi\alpha'$ L = length $= V_i - V_j$

These strings are charged under the $U(1)$ gauge fields living in each brane, one end of string has charge $+1$ & other end carries charge -1. Charges $\pm(1,-1)$ under $U(1)_i \otimes U(1)_j$

States with $N=1$: mass $m^2 = \left(\frac{V_i - V_j}{2\pi\alpha'}\right)^2$

$\alpha_{-1}^{25} \, |p; ij\rangle$ $i,j = 1 \ldots n$

$\alpha_{-1}^{\mu} \, |p, ij\rangle$

$\longrightarrow$ n^2 scalars

$\longrightarrow$ n^2 gauge fields

for $V_i \neq V_j$ only those states with $i=j$ are massless. n massless gauge fields, $U(1)^n$, & n massless scalars $\longrightarrow$ n, X^{25} D brane locations

Taking all V_i equal, get n^2 massless gauge fields & n^2 massless scalars.

$U(1)^n \longrightarrow U(n)$, ij stretched strings $\rightarrow W$ bosons become massless when branes coincide. Separating the branes = Higgs mechanism. $X = n \times n$ adjoint scalar $\langle X \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_n \end{pmatrix}$ breaks $U(n) \longrightarrow U(1)^n$ for v_i all different.

Above gives interesting correspondence between D branes & gauge theories living in their world volume.

D brane tension $\sim$ mass/Area $\sim \dfrac{1}{g_s \, l_s^{p+1}}$ $\quad (l_s^2 \sim \alpha')$

$g_s \sim$ string coupling

e.g. D0 brane $\sim$ particle has mass $m \sim 1/g_s l_s$.

Solitons often have masses $\sim 1/g_s^2$

D branes are like solitons, but with mass $\sim 1/g_s$

For small g_s, D branes very heavy or tensionful. D0 brane has mass $\sim \frac{1}{g_s l_s}$

Suggests it can probe distances $\sim g_s l_s$

= smaller than string length for small g_s

$\xrightarrow{?}$ "more fundamental than strings"? Strings

made of D0 branes? Note N particles

in D dimensions have $N \cdot D$ coordinates,

but N D0 branes in D dimensions have

$N^2 \cdot D$ coordinates since $X^\mu \to N \times N$ matrices.

New directions are very massive when D0s

are separated, recover ND coordinates in

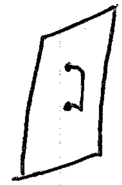
this limit. But at small distances (separations)

get new degrees of freedom, massless & on

same footing as usual coordinates (related

by a symmetry).

There are interesting connections between D branes & black hole horizons.



← string with ends on D brane ~ Closed string 1/2 hidden behind horizon. Find

$$S_{BH} = \frac{k_B \text{ Area}}{4\, l_p^{d-2}}$$

reproduced by the entropy of the gauge fields & matter which are confined to live on the D brane. Gives microscopic description of black hole entropy as degrees of freedom living on the horizon ~ D brane, with ~ 1 bit of information per planck area.

Also recent "phenomenological" interest in scenarios where we live in a D3 brane floating in extra dimensions (~ flatland). Gravity & other closed string modes can leak into extra dimensions ⟶ corrections to $1/r^2$ gravity force law at small distances ~ $1/r^{D-2}$.

Up to now, we've been discussing the "bosonic string". All states in physical spectrum are bosons, e.g. tachyon, gauge field, graviton, etc. only integer spin. Also problematic because of tachyon (maybe). "Superstrings" cure these ~~the~~ problems; contain fermions in spectrum & no tachyon. Idea: replace spacetime vectors = worldsheet scalars $X^\mu \to (X^\mu, \psi^\mu)$ where ψ^μ = worldsheet fermion (& spacetime vector)

Make worldsheet theory supersymmetric, symmetry 2d worldsheet ~~scalars~~ bosons $\longleftrightarrow$ fermions.

$$X^\mu \longleftrightarrow \psi^\mu$$

h_{ab} metric $\longleftrightarrow$ $\chi_{a\alpha}$ worldsheet spin 3/2 "gravitino"

$\frac{\delta}{\delta h} \longrightarrow T(z)$ stress tensor $\qquad \frac{\delta}{\delta \chi} \longrightarrow G(z)$ spin 3/2 supercurrent

$\hookrightarrow$ leads to spacetime Klein Gordon eqn for bosons

$\hookrightarrow$ leads to spacetime Dirac eqn for fermions

$$C_M = D\Big(\underset{X^\mu}{1} + \underset{\psi^\mu}{\tfrac{1}{2}}\Big) \qquad C_{\text{Ghosts}} = \underset{\text{bc ghosts}}{-26} + \underset{\text{superpartners}}{11}$$

$C_M + C_{\text{ghost}} = 0$ for $\underline{D_{crit} = 10}$.

Various $D = 10$ superstrings:

Type IIA, Type IIB: closed strings (but includes D branes) differ in a relative sign choice for left vs right movers. IIB theory is chiral in spacetime.

Type I: includes open strings, $SO(32)$ worldsheet global / spacetime gauge symmetry (req'd by $SL(2,\mathbb{Z})$ modular invariance of partition fn).

Heterotic $E_8 \times E_8$ or $SO(32)$: Worldsheet supersymmetry only for left movers z, $\bar{z}$ nonsupersymmetric. $D_L = 10$. $D_R = 26$. Extra 16 right moving "dimensions" are compact with worldsheet global / spacetime gauge symm $SO(32)$ or $E_8 \times E_8$ (req'd by modular inv.) These are <u>chiral</u>.