Usual particle physics: elementary particles are points

e^- $\cdot$ ∞'ly density of mass & charge $\delta(\vec{x}-\vec{x}(t))$

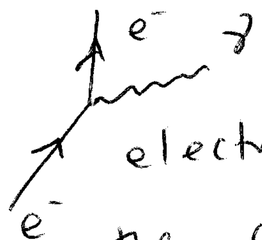
(QM: a bit fuzzy, but still..)

<u>Strange</u> eg $\overset{\circ}{e^-} \xleftrightarrow{R} \overset{\circ}{e^-}$ $\quad V \sim \dfrac{e^2}{R}$ $\quad$ take $R \to 0$, $V \to \infty$

more energy than in entire universe could be stored in two very near electrons.

<u>Can't happen</u> in gravity thy $\to$ <u>forms black hole</u>

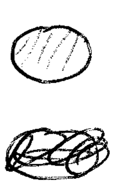
Also a point like electron by itself would really be a black hole $R = 2GM$. What does this then mean?:



photon interacts with electron but how could it touch the electron? Once within $R \leq 2GM$, inside horizon, could never escape!

Counter intuitive anyway to have point like particles.

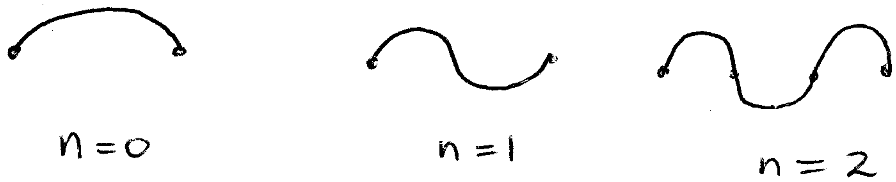
replace $\cdot \longrightarrow \sim$ or $\bigcirc$ strings?



membranes?

or blob?

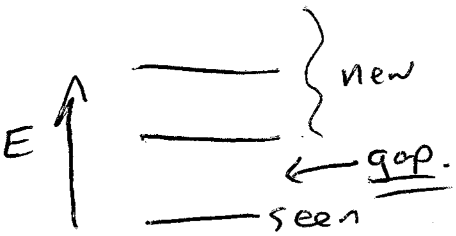
Only strings work. Quantize $\longrightarrow$ new particles

String 

etc.

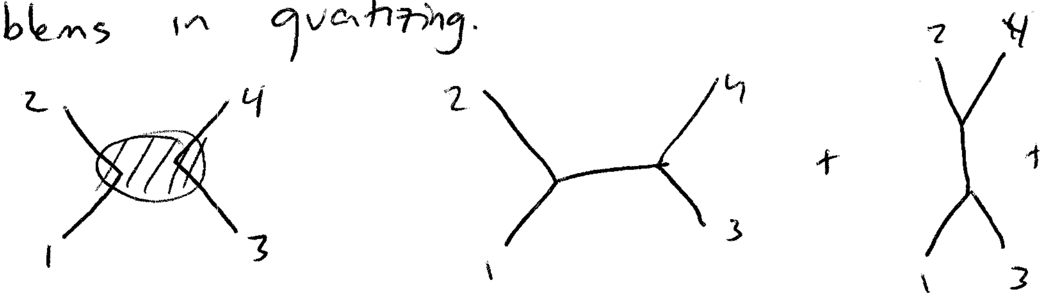
harmonics. $n = 0$ light → known particles

$n > 0$ massive, $m^2 \sim n T$
string tension



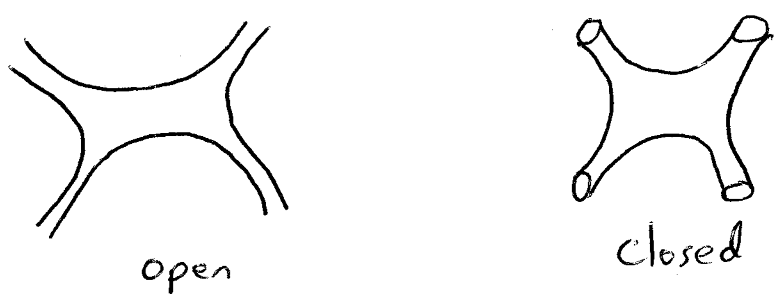
Membrane turns out continuous spectrum 
of particle masses. Doesn't agree w/ observation.

Other problems in quantizing.

Particles 

Strings

<u>smooth</u>



Very <u>soft and fluffy</u> at high energies.
Softer even than asymptotic freedom of gauge theories

Bad UV behavior gone!

Nice: Only known consistent thy of quantum gravity

(Clash between QM & GR)

Strange: Extra dimensions, ∞ number of new

massive particles. Top down must work to get massless

particles to resemble our world ~~[illegible]~~

(not hard to get gravity & other forces,

some trouble with extra light scalars).

Why bother? Forces: Gravity $\rightarrow$ GR (classical)

Strong, Weak, EM } QFT Standard Model

firewall?

QED & Standard model pretty good

eg. g-2 of electron compute & check w/ exp to

1 part in 10^{-12}!

In standard model ignore gravity. It's weak, right?

E & M $V(r) = \frac{e^2}{r}$ + corrections

dim'l analysis $\hbar = c = 1$ $E \sim m \sim 1/L$

$e \sim 1$ dim'less $e^2/4\pi\hbar c = 1/137$

$m \xleftrightarrow{r} m$ $V = -\frac{Gm^2}{r} \rightarrow G \sim 1/m^2$

$G_N \equiv 1/M_P^2 \equiv \ell_P^2$ Planck mass/length scale

$M_P = 1.2 \times 10^{19}$ GeV $\quad \ell_P = 1.6 \times 10^{-33}$ cm

$G_{eff} = E^2/M_P^2$ large for $E \sim M_P$

New gravity physics up there.

Similar to weak interactions $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

coupling $G_F \sim 1/M_W^2$ new physics

@ $M_{W,Z} \sim$ ~~...~~ ~~...~~ 90 GeV in standard model.

Our accelerators now probe only to

$\sim 10^{-16}$ cm, far from $\ell_P \sim 10^{-33}$ cm

But gravity & other forces are <u>not just</u>

<u>forces</u> - they're <u>symmetry principles</u>.

<u>GR</u>: Gravity $\longleftrightarrow$ General covariance

$x^\mu \rightarrow x^{\mu'} = x^{\mu'}(x^\mu)$ symm.

Like saying don't bother to make thy relativistic inv just because $v \ll c$. Recall making QM compatible w/ special relativity ~~...~~ ~~...~~ $\rightarrow$ new conceptual framework QFT: $\rightarrow$ antimatter, particles created and annihilated, etc.

<u>How to Quantize w/ Gen Cov?</u>

Many conceptual confusions with gravity. Eg black holes & quantum information problem. Holography.

horizon: $R = 2M\ell_p^2$ $\qquad k_B T = 1/8\pi M \ell_p^2$

1st law $\quad dE = TdS$ with $E = M$.

* Show $S \equiv S_{BH} \equiv k_B \dfrac{\text{Area}}{4\ell_p^2}$ ← of horizon is sol'n

2nd law increase of entropy $\Longrightarrow$

$$S \le S_{BH} = k_B \frac{\text{Area}}{4\ell_p^2}$$

Area surrounding any region

entropy inside area A

Always!

Note not ~ volume! Holographic storing of info.

What states counted?

Also information problem: pure $\xrightarrow{\text{(Hawking claim)}}$ mixed states??

Throw information in pure state into black hole, which then radiates away at temp T, until it evaporates. Is information gone? Encoded in radiation? (Hawking says no)

Can happen everywhere via tiny virtual black holes...

All forces $\longleftrightarrow$ local "gauge" symmetries

Eg E & M $\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\partial^\mu F_{\mu\nu} = J_\nu$$

$A_\mu \to A_\mu + \partial_\mu f \qquad \psi \to e^{-iqf}\,\psi$

f = arbitrary function $f(x_\mu)$. Replace ∂_μ with $D_\mu = \partial_\mu + ie A_\mu$

$$S = \int d^4x \left(-\frac{1}{4e^2} \underbrace{F_{\mu\nu} F^{\mu\nu}}_{\vec{E}^2 - \vec{B}^2} + \mathcal{L}_{matter} \right)$$

all derivaties are $D_\mu = \partial_\mu + ie A_\mu$

Vary $A_\mu \to A_\mu + \delta A_\mu$, $\quad S \to S + \delta S$

$$\delta S = \int d^4x \left(\frac{\delta \mathcal{L}_{matter}}{\delta A_\mu} \right) \delta A_\mu \equiv \int d^4x\, J^\mu \delta A_\mu$$

symmetry under $\delta A_\mu = \partial_\mu f \Rightarrow \delta S = 0$

$\Rightarrow \partial_\mu J^\mu = 0 \qquad$ current conservation

Charge quant $q = n e_0 \Rightarrow \psi \to e^{iqf}\psi$ invt under $f \sim f + 1/e_0 \Rightarrow f$ lives on a <u>Circle of radius $1/e_0$</u>. $U(1)_{EM}$ is symm. of this circle. What is this mysterious & abstract circle? Is it really there?

G.R.: symm acts on our actual observed space, general coord. transfs $X^\mu \to X^{\mu'}(X^\mu)$

Seems more obvious & physical than the f "coordinate" of E & M.

$$S = \int d^D x \sqrt{\det g} \left(\frac{-R}{16\pi G_N} + 2\Lambda + \mathcal{L}_{matter} \right)$$

$$R = g^{\mu\nu} g^{\rho\sigma} R_{\mu\rho\nu\sigma} \quad \text{Ricci Scalar}$$

$$R_{\mu\rho\nu\sigma} \equiv \frac{1}{2} \left[\frac{\partial^2 g_{\mu\nu}}{\partial x^\rho \partial x^\sigma} - (\mu \leftrightarrow \rho) - (\nu \leftrightarrow \sigma) + (\mu \leftrightarrow \rho \text{ and } \nu \leftrightarrow \sigma) \right] + g_{\kappa\lambda} \left[\Gamma^\kappa_{\mu\nu} \Gamma^\lambda_{\rho\sigma} - \Gamma^\kappa_{\rho\nu} \Gamma^\lambda_{\mu\sigma} \right]$$

$$\Gamma^\kappa_{\mu\nu} \equiv \frac{1}{2} g^{\kappa\sigma} \left(\frac{\partial g_{\sigma\mu}}{\partial x^\nu} + \frac{\partial g_{\sigma\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right)$$

connection.

Covariant derivatives eg.

$$\left\{ \begin{array}{l} D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\kappa} V^\kappa \\ D_\mu V_\nu = \partial_\mu V_\nu - \Gamma^\kappa_{\mu\nu} V_\kappa \end{array} \right.$$

Under $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$, $S \to S + \delta S$

$$\delta S = \int d^D x \sqrt{\det g} \left(\tfrac{1}{2} T^{\mu\nu} \, \delta g_{\mu\nu} \right)$$

(gets contributions from gravity & $\mathcal{L}_{\text{matter}}$)

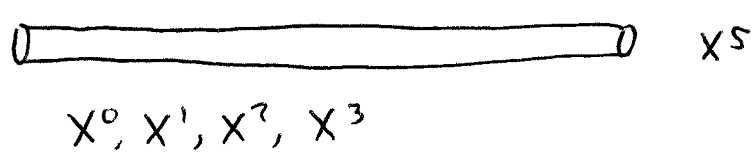
General coord invariance of δS eg

under $\delta g_{\mu\nu} = D_\mu V_\nu + D_\nu V_\mu$

with V_μ arbitrary vector function $V_\mu(X^\mu)$

$\Rightarrow D_\mu T^{\mu\nu} = 0$ Conserved stress tensor $\Rightarrow$ Conserved charges = Energy & momentum.

Kaluza ('1921) & Klein



X^5 a circle of radius $R = l_p / e_0$

$$ds^2 = g_{\mu\nu} \, dx^\mu dx^\nu + e^{2\phi} (dx^5 + l_p A_\mu dx^\mu)^2$$

$$\left.\begin{array}{l} X^5 \to X^5 - l_p f(X^\mu) \\ A_\mu \to A_\mu + \partial_\mu f \end{array}\right\}$$ Subgroup of 5d general coord inv = 4d E&M gauge inv.

5d graviton $\to$ 4d graviton + photon + scalar

5d Ricci Scalar $R^{(5d)} \to R^{(4d)} +$

$-\frac{1}{4} e^{2\phi} F_{\mu\nu} F^{\mu\nu} - 2e^{-\phi} \nabla^2 e^{\phi}$

for A_μ & ϕ taken to be indep of X^5.

So 5d Einstein Hilbert action $\to$ 4d Einstein Hilbert + Maxwell action (with extra "Brans Dicke" coupled scalar ϕ)

Field $\varphi(X^\mu, X^5)$ in X^5 momentum eigenstate

$\varphi(X^\mu, X^5) = e^{i p_5 X_5} \varphi(X^\mu)$

Under $X^5 \to X^5 - l_P f$, $A_\mu \to A_\mu + \partial_\mu f$

$\varphi \to e^{-i p_5 l_P f} \varphi \implies \varphi$ has $U(1)_{EM}$

electric charge $q = p_5 l_P$. P_5 momentum conservation $\Rightarrow$ electric charge conservation

$J_\mu = T_{\mu 5}$. P_5 is quantized by

$X^5 \sim X^5 + 2\pi (l_P/e_0) \implies q = p_5 l_P = n e_0$

Charge quantization explained!

Each 5d field $\varphi(X^\mu, X^5) \Rightarrow$ infinite tower of 4d fields.

$$\varphi(X^\mu, X^5) = \sum_{n=-\infty}^{\infty} e^{\frac{in\ell_0}{\ell_p}} \varphi_n(X^\mu).$$

If 5d field φ has mass M, 4d field φ_n has mass m_n given by

$$m_n^2 = M^2 + \left(\frac{n e_0}{\ell_p}\right)^2 \leftarrow (X^5 \text{ spacelike})$$

$$m_n^2 = M^2 - \left(\frac{n e_0}{\ell_p}\right)^2 \leftarrow (X^5 \text{ timelike})$$

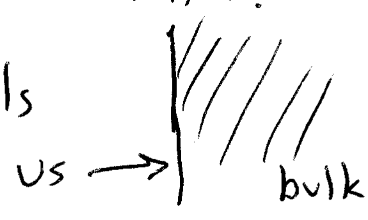
* Show why this is true.

$\Rightarrow$ Need to have X^5 spacelike!

(Otherwise infinite tower of tachyons with $m_n^2 < 0$.)

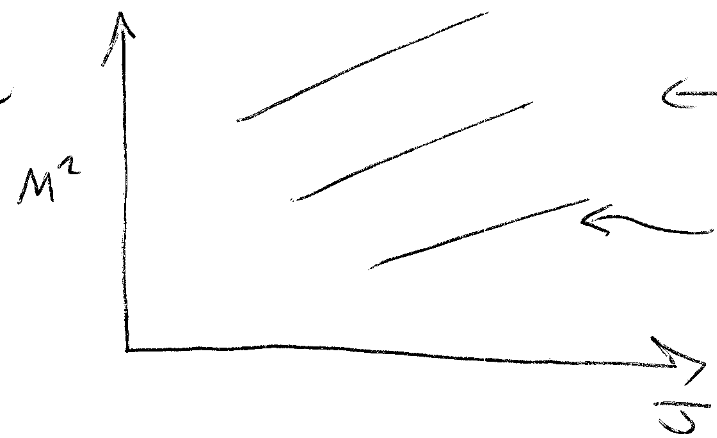
Can also try to get $SU(3) \times SU(2) \times U(1)$ via extra dimensions (since compact groups!)

But hard to get chiral matter.

(Only way via domain walls 

)

More history strings came from strong ints.

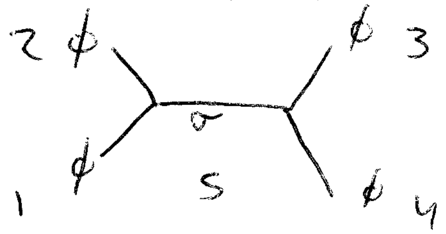


$\leftarrow M^2 \geq (J+a)/\alpha'$

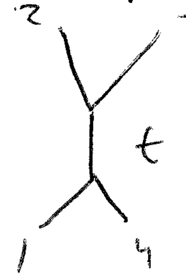
$\leftarrow M^2 = (J+a)/\alpha'$

$\alpha' \sim M_{css}^2$

Fundamental?



General problems with spin > 1



$$s = -(p_1+p_2)^2$$
$$t = -(p_2+p_3)^2$$
$$u = -(p_1+p_3)^2$$

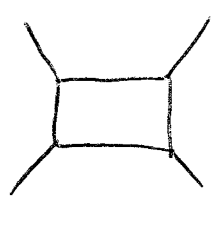
$$s+t+u = \sum m_i^2$$

$$\mathcal{L} = g\,\phi^* \partial_{\mu_1} \cdots \partial_{\mu_J} \phi\, \sigma^{\mu_1 \cdots \mu_J}$$

consider t channel: $$A_J(s,t) = -\frac{g^2 (-s)^J}{t - M^2}$$

bad for large s   worse for larger J

J = 1 just ok in 4d



$$\sim \int \frac{d^D p}{(p^2)^2} A^2 \sim \text{log diverg for } J=1$$

$$A(s,t) = -\sum_J \frac{g_J^2 \, (-s)^J}{t - M_J^2}$$ make ∞ sum

recall $e^{-x} = \sum$ better large x behavior than ~~any~~ each term.

Venizano amplitude ('68)

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$ (Beta fn)

$$\alpha(s) = \alpha(0) + \alpha' s$$

Duality: $A(s,t) = A(t,s)$

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)(\alpha(s)+2)\cdots(\alpha(s)+n)}{n!} \frac{1}{\alpha(t)-n}$$

poles at $t = M^2$ for $M^2 = (n-\alpha(0))/\alpha'$

residue $\sim$ $s^n + s^{n-1} + \cdots + 1$ $\leftarrow$ spin $J \leq n$

$$A(s,t) = \int_0^1 dx \, x^{-\alpha(s)-1} (1-x)^{-\alpha(t)-1}$$

Large s fixed t, high energy small $\measuredangle$ scattering

Use $\Gamma(u) \simeq \sqrt{2\pi}\, u^{u-1/2} e^{-u}$ for $u \to \infty$

$$A(s,t) \sim \Gamma(-\alpha(t))\,(-\alpha(s))^{\alpha(t)}$$

$$\sim s^{\alpha(t)} \quad \text{vs} \quad s^J \text{ for particles.}$$

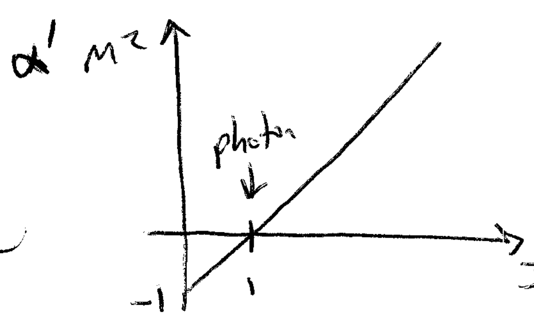
In physical region of elastic scattering

$t<0$, eg fixed $\measuredangle$ $s \to \infty$ $t \to -\infty$
s/t fixed

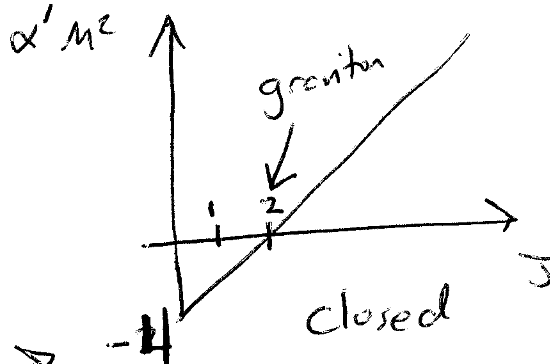
$$A(s,t) \sim s^{\alpha(t)} \ll s^J$$

Softer than any field thy. Even asymp. freedom.
Soft & fluffy strings.

No ghosts $\longrightarrow$ $\alpha(0)=1$ $D=26$
to get $g^2>0$ right residue sign

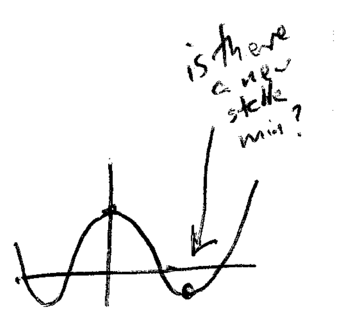


open



closed

tachyons - bad, unstable vacuum eg.



World line of particles $(- + + + \cdots +)$

$$S_{pp} = -m \int d\tau \left(- \dot{X}^\mu \dot{X}_\mu \right)^{1/2} \qquad \dot{} = \frac{\partial}{\partial \tau}$$

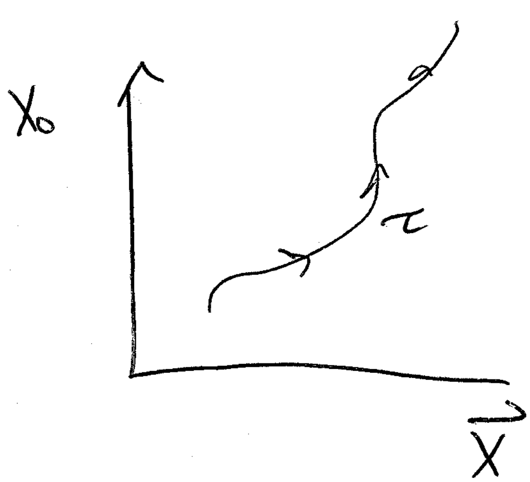
* show in non rel limit $S \approx \int dt \left(T - V \right)$

with $T = \frac{1}{2} m \left(\frac{d\vec{x}}{dt} \right)^2$, $V = m$

- S_{pp} EOM: $\dot{u}^\mu = 0$ $\qquad U^\mu \equiv \dot{X}^\mu \left(-\dot{X}^\nu \dot{X}_\nu \right)^{-1/2}$ = velocity
- S_{pp} invt under $\tau \longrightarrow \tau' = \tau'(\tau)$ general coord transf.

$$S = \frac{1}{2} \int d\tau \left(\eta^{-1} \dot{X}^\mu \dot{X}_\mu - \eta m^2 \right)$$

$\eta \sim \sqrt{h}$ metric of τ worldline



$$\eta'(\tau') d\tau' = \eta(\tau) d\tau$$

$$\eta \text{ EOM}: \ \eta^{-2} \dot{X}^\mu \dot{X}_\mu = -m^2$$

For $m \neq 0$ solve for η

$$\eta^2 = -\dot{X}^\mu \dot{X}_\mu / m^2$$