215c, 5/20/20 Lecture outline. © Kenneth Intriligator 2020.

⋆ **Week 8 reading: Tong chapter 5 up to WZW term. Start Tong chapter 3 on anomalies.**

http://www.damtp.cam.ac.uk/user/tong/gaugetheory.html

• Last time: $SU(N_c)$ with N_f massless Dirac Fermions has a $G = SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$ classical global symmetry. The $U(1)_A$ is axial, and anomalous. In the broken phase, the operator $\tilde{\psi}\psi$ gets a bose condensate, which spontaneously breaks $G \to H$ with $H = SU(N_f)_D \times U(1)_V$. The $U(1)_V$ is baryon number symmetry. The LEEFT consists of $\pi^a \in G/H = SU(N_f)_{L-R}$, which only have derivative interactions and constant shifts are the secret $SU(N_f)_{L-R}$ symmetry (before adding mass terms that explicitly break $SU(N_f)_{L-R}$. The global $U(1)_V$ is baryon number symmetry, but the π^a are all neutral. The charged baryons come from solitons in the pions, associated with $\pi_3(G/H) = \mathbf{Z}$; these are called Skyrmions. It is a highly non-trivial and nice check that the solitons indeed have the right quantum numbers to match with the baryons. For example, since the baryon is made from N_c spin $\frac{1}{2}$ Fermions, contracted with an epsilon tensor in the gauge indices, it must be symmetric in the remaining $SU(N_f)_D$ and spin $SU(2)_D$, with $(-1)^F = (-1)^{N_c}$. The global flavor quantum numbers rely on properly quantizing the collective coordinates, and it only works thanks to the WZW term.

The chiral Lagrangian has both parity $P_0 : \vec{x} \to -\vec{x}$ symmetry and $U \to U^\dagger$ symmetry, i.e. $(-1)^{N_\pi}$. The correct parity of the UV QCD theory is $P = P_0(-1)^{N_\pi}$. Sometimes the LEEFT has an extra symmetry, called accidental, that is not there in the underlying theory. This is not such a case. The apparent extra symmetry is an artifact of missing an interaction term, the WZW term, which separately breaks P_0 and $(-1)^{N_\pi}$. It cannot be written as an integral over a 4d Lagrangian, but instead requires going to a 5d space Y with our spacetime as its boundary, $X_{4d} = \partial Y$. The WZW term is $S_{WZW} = k\int_Y \omega_5$, where ω_5 is the volume 5-form on G/H, $\omega \sim \mathrm{Tr}(U^\dagger dU)^{\wedge 5}$. The 5d action is not a total derivative, so the answer for different Y's generally differs. The difference is $\int_{Y-Y'\cong S^5} \omega_5 = 2\pi\pi_5(S^4) \in 2\pi\mathbf{Z}$ since ω is normalized to give the S^5 winding number. Then $e^{ik\int_Y \omega_5}$ is invariant under this ambiguity if $k \in Z$. Witten showed that everything works correctly, including anomaly matching and getting the right quantum numbers of the baryon from the skyrmionic soliton, if $k = N_c$.

• If $U(1)_A$ were a symmetry, there would have to be a 9th pseudoscalar (since it is P odd) meson; the candidate observed particle is called the η', but it is too massive to be considered an approximate NGB. Estimates for its mass based on the quark masses suggested $m_{\eta',wrong} \approx 355MeV$ whereas $m_{\eta',actual} \approx 958MeV$. The resolution is that $U(1)_A$ is not a symmetry, because it has a quantum anomaly, and this gives the η' a large mass compared to the light pions.

Before discussing anomalies in more detail, let's mention another related puzzle: the $\pi^0 \to \gamma\gamma$ decay lifetime. While $\pi^\pm$ are relatively long-lived $\sim 10^{-8}s$ ($\pi^+ \to \bar{\mu} + \nu_\mu$), as expected for a NGB, the π^0 has a short lifetime of $10^{-16}s$. The decay proceeds by an interaction term in the LEEFT that is familiar from our discussion of instantons, axions, and a previous HW exercise: $\mathcal{L} \supset g\pi^0\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$. This is consistent with the fact that π^0 is a parity-odd pseudoscalar, but it is inconsistent with π^0 being a true NGB: it is not really a derivative coupling (well, we can integrate by parts to write it as $\partial_\mu\pi^0K^\mu$ but K^μ is not gauge invariant). The coupling leads to a decay rate $\Gamma(\pi^0 \to 2\gamma) = m_\pi^3 g^2/\pi$. A naive estimate of g is that it is generated by a loop, so $g_{naive} \sim \alpha/2\pi f_\pi$; this leads to $\Gamma_{naive} \sim \times 10^{16}s^{-1}$, which fits well with observation. But a seemingly less naive estimate is that this decay involves a NGB, which suggests that it is requires an extra factor of the $SU(2)_{L-R}$ breaking scale $m_u + m_d \sim m_\pi^2$, which should be $\sim m_\pi^2/m_N^2$, leading to $\Gamma_{\text{“}improved\text{”}} \sim 10^{13}s^{-1}$, which does not fit with observation. This was connected to the triangle diagram anomaly in 1969 by Bell and Jackiw, and Adler. As a low-energy model for how this happens, consider QED with an electrically charged Fermion ψ (which can be a nucleon, e.g. the proton, containing up and down quarks), with a pseudoscalar π^0 that couples to the Fermion via $\mathcal{L} \supset i\lambda\pi^0\bar{\psi}\gamma^5\psi$. A loop with π^0 and λ at one vertex, and photons at the other two, leads to the coupling with $g = \lambda\alpha/2\pi m$, with $\lambda = m/f_\pi$ and the mass m of the Fermion cancels out. The loop diagram is essentially the anomaly triangle diagram, to be discussed soon.

• Next topic: spontaneous breaking gauge symmetries and the “Higgs ”(+ Anderson, Brout, Englert, Goldstone, Guralnik, Hagen, Kibble, Polyakov, Migdal, Schwinger, 't Hooft) mechanism. Recall the simple example of the Abelian Higgs model: a complex scalar field ϕ that is charged under a $u(1)$ gauge symmetry: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|)$ with $D_\mu\phi = (\partial_\mu + ieA_\mu)\phi$. We take e.g. $V = -\frac{1}{2}m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$, so the vacuum is at $\langle|\phi|\rangle = v/\sqrt{2} = \sqrt{2m^2/\lambda}$. Write $\phi(x) = \frac{v+h(x)}{\sqrt{2}}e^{i\pi(x)/f_\pi}$ and plug back in and expand find, $\mathcal{L} \to -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2(A_\mu + \frac{1}{ef_\pi}\partial_\mu\pi)^2 + \ldots$. Gauge invariance forbids ordinary masses, but here we see a loophole: the gauge field has mass $m_A = ev$ and gauge transformations $A_\mu \to A_\mu + \partial_\mu\alpha$ are accompanied by the familiar pion shifts associated with secret symmetry, $\pi(x) \to \pi(x) - ef_\pi\alpha(x)$. Phrased this way, it's also called the Stueckelberg mechanism.

But now we can use gauge invariance to choose $\alpha(x)$ to set $\pi(x) \to 0$; this is called unitary gauge. The gauge field A_μ has become massive, meaning that it has 3 polarizations, where the longitudinal polarization comes from "eating the Goldstone boson".

• This same mechanism occurs in BCS theory, where the condensate of Cooper pairs breaks $U(1)_{E\&M} \to \mathbf{Z}_2$. The massive photon leads to superconducting phenomena, like persistence of currents and the Meissner effect, where magnetic flux is confined to thin flux tubes. This is because the energy is minimized by $A_\mu \sim \partial_\mu \pi$ pure gauge, so $F_{\mu\nu} \to 0$. Electric charges are screened, and magnetic flux is confined. Jorge Hirsch has been interested in making the mechanism more explicit. Note that in an electric-magnetic dual version, replacing $F_{\mu\nu} \to \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, having a condensate of magnetic monopoles would lead to electric flux confinement. It had been a longstanding hunch that something similar could explain color flux confinement in QCD, and this has been shown to work in supersymmetric theories.

• Schwinger understood back in the 1950s how current algebra correlation functions can be modified to yield the propagator for a massive gauge field – that is a current algebra version of the photon getting a mass by eating the NGB. Write $\langle TA_\mu(x)A_\nu(y)\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} iG^{\mu\nu}(p)$ where, e.g. in Feynman gauge, $iG^{\mu\nu} = \frac{-ig^{\mu\nu}}{p^2+i\epsilon} + \mathcal{O}(e^2)$. Summing the geometric series of 1PI corrections can lead to $iG^{\mu\nu} = -\frac{ig^{\mu\nu}}{p^2(1-e^2\Pi_2(p^2))} + \ldots$, where the 1PI diagrams lead to $\langle J^\mu(p)J^\nu(-p)\rangle = (g^{\mu\nu} - p^\mu p^\nu)\Pi_2(p^2)$. The gauge field can get a mass m_A if $e^2\Pi(2(p^2) = \frac{m_A^2}{p^2} + \ldots$. We discussed in an earlier lecture how SSB means that $J^\mu(x)$ creates the particle $\pi(x)$ from the vacuum, and how that leads to a $1/p^2$ pole in the current two-point function spectral density from the massless particle.