215c Homework exercises 3, Spring 2020, due April 24

1. Please explicitly verify the following statements from lecture:

   (a) Plugging the gauge transformation $A_\mu^U = U(i\partial_\mu + A_\mu)U^{-1}$ into $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ leads to $F_{\mu\nu}^U = UF_{\mu\nu}U^{-1}$.

   (b) Using $D_\mu F_{\rho\sigma} = \partial_\mu F_{\rho\sigma} - i[A_\mu, F_{\rho\sigma}]$ and the above expression for $F_{\mu\nu}$ in terms of A_μ and A_ν ensures the Bianchi identity $D_\mu F_{\rho\sigma} + (cyclic) = 0$. Relate this to, or separately verify, the Jacobi identity for covariant derivatives $[D_\mu, [D_\rho, D_\sigma]] + (cyclic) = 0$.

   (c) Verify in detail (without using the EOM) that $D_\nu(D_\mu F^{\mu\nu}) = 0$.

2. Let ϕ^c be a complex scalar field in the fundamental representation of $su(2)$, with $c = 1, 2$.

   (a) Write down the general form of a Lorentz invariant Lagrangian density $\mathcal{L}$ that is quadratic in the ϕ^c fields and $su(2)$ gauge invariant. Please write everything out in detail.

   (b) Write out in detail the contribution to the covariantly conserved $su(2)$ currents, which couple linearly to the $su(2)$ gauge fields, coming from the ϕ^c matter fields.

   (c) Suppose that that the scalar field has a Bose condensate, $\langle\phi\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$ where v is a constant vev and the column vector is in $c = 1, 2$ color space. Argue that any non-zero vev could be rotated to this form, with v real, by a gauge transformation.

   (d) Let $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix} + \phi'$ and write down all the terms in the above $\mathcal{L}$ that involve v.

3. In the case of $u(1)$ gauge theory with a charged scalar ϕ, you know how to write down the Feynman propagators and vertices. The modification for the above case is that the scalar and gauge fields have additional color indices, which show up as additional labels on the propagators and vertices. Write down the momentum space Feynman propagator for the scalar fields, and also their interaction vertices involving the gauge fields, coming from the $\mathcal{L}$ of the previous question (taking $v = 0$).