215c Homework exercises 2, Spring 2020, due April 15

1. Replace the θ coupling with a dynamical axion field $\theta \to ga(x)$, where g is a coupling constant and a is a canonically normalized scalar field.

   (a) Derive the momentum-space Feynman rule for the $a\gamma\gamma$ interaction vertex.

   (b) Compute to lowest order amplitude for the decay of the axion to two photons. Allow the axion and the photons to be off-shell.

   (c) Compute the total inclusive amplitude squared for the above process, and verify that it vanishes if the axion and photons are massless and on-shell.

2. Consider a theory of N complex scalar fields $\phi_{i=1\ldots N}$, with $\mathcal{L} = \sum_{i=1}^{N} |\partial_\mu \phi_i|^2 - f(\sum_i |\phi_i|^2)$.

   (a) What is the global symmetry, and write down the associated conserved currents.

   (b) Suppose now that all N scalars are charged under a $u(1)$ gauge symmetry, all with charge $+1$. Now what is the global symmetry? Write down the Feynman rules for the photon's interaction(s).

3. Consider QED in $2+1$ dimensions, taking the Fermion to be 2-component, with Dirac matrices $\gamma^0 = \sigma_3$, $\gamma^1 = i\sigma_1$, $\gamma^2 = i\sigma_2$, with σ_i the Pauli matrices.

   (a) Compute $\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\sigma)$. Note that it is parity odd (define parity in a sensible way, so it is not just a rotation).

   (b) Note that the mass m of the Fermion is real, and that it violates parity.

   (c) Write down the one-loop contribution to the photon self-energy $\Pi_{\mu\nu}(k)$ (two-point function with amputated external propagators) in terms of a Feynman loop integral and a trace in Dirac space. Note that it contributes a term $\Pi_{\mu\nu} = \ldots + i\epsilon_{\mu\nu\lambda} k^\lambda \Pi_2(k^2)$, writing $\Pi_2(k^2)$ as a Feynman loop integral. You do not need to do the loop integral.

   (d) Note (just from staring at the integral – it's extra credit if you evaluate it) that $\Pi_2(0)$ is dimensionless, and that it picks up a minus sign under $m \to -m$. Note that it corresponds to inducing a non-zero Chern-Simons term in the low-energy theory upon integrating out a Fermion of mass m.