4/9/19 Lecture outline

$\star$ Reading: Zwiebach chapters 2 and 3.

• Last time: Statement of relativity principle: physics is indistinguishable among all inertial frames. If one frame is inertial, all frames moving relative to that at a constant velocity are also inertial. Related by $x^{\mu'} = \Lambda^{\mu'}_{\nu} x^{\nu}$ where $\Lambda^T \eta \Lambda = \eta$, where η is the metric, chosen to have mostly plus convention. All 4-vectors transform the same way: $b^{\mu'} = \Lambda^{\mu'}_{\nu} b^{\nu}$. A tensor transforms as $T^{\mu'\nu'} = \Lambda^{\mu'}_{\kappa} \Lambda^{\nu'}_{\sigma} T^{\kappa\sigma}$. Can contract or raise or lower indices with $\eta_{\mu\nu}$, e.g. $a^{\mu} b^{\nu} \eta_{\mu\nu} = -a^0 b^0 + \vec{a} \cdot \vec{b}$ and $T^{\mu}_{\mu} = -T^{00} + \delta_{ij} T^{ij}$ are 4scalars. 4-scalars are invariant under Lorentz transformation. The proper time element $d\tau = \sqrt{-dx^{\mu} dx_{\mu}/c^2}$ is a 4-scalar. $u^{\mu} = dx^{\mu}/d\tau$ is the velocity 4-vector, and $d^2 x^{\mu}/d\tau^2$ is the acceleration 4-vector. The statement of relativity requires that all formulas relate quantities that transform the same way, for example $\vec{F} = m\vec{a}$ can fit with relativity if it becomes a 4-vector equation: $f^{\mu} = m dp^{\mu}/d\tau$ (the time component here relates power to the time derivative of energy).

$p^{\mu} = (E/c, p_x, p_y, p_z)$, with $p_{\mu} p^{\mu} = -m^2 c^2$. p^{μ} transforms as a Lorentz 4-vector, $p^{\mu'} = \Lambda^{\mu'}_{\nu} p^{\nu}$. Proper time: $ds^2 = c^2 dt_p^2 = c^2 dt^2 (1 - \beta^2)$. $u^{\mu} = c dx^{\mu}/ds = dx^{\mu}/dt_p = \gamma(c, \vec{v})$, and $u_{\mu} u^{\mu} = -c^2$. A massive point particle has $p^{\mu} = m u^{\mu}$. Massless particles, like the photon, have p^{μ} with $p^{\mu} p_{\mu} = 0$.

• Quantum mechanics: replace $p^{\mu} = (H/c, \vec{p}) \to -i\hbar \partial^{\mu}$. Free particle wavefunction $\psi \sim \exp(ip \cdot x/\hbar)$; $p_{\mu} x^{\mu} \equiv p \cdot x$ is Lorentz invariant.

• Light cone coordinates: $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The bad: spoils rotational symmetry. The good: will make it easier to quantize string theory (avoids having to discuss here more advanced alternatives, which do not require going to the light cone coordinates). $-ds^2 = -2dx^+ dx^- + dx_2^2 + dx_3^2 = \widehat{\eta}_{\mu\nu} dx^{\mu} dx^{\nu}$. $a_{\pm} = -a^{\mp}$. Take $p^{\pm} = \frac{1}{\sqrt{2}}(p^0 \pm p^1) = -p_{\mp}$. So $i\hbar \partial_{x^+} \to -p_+ = E_{lc}/c$, i.e. $p^- = E_{lc}/c$.