4/4/19 Lecture outline

$\star$ Reading: Zwiebach chapters 1 and 2.

• Last time: Quantum field theory is one of humankind's most accurately tested theories! E.g. the magnetic moment of the electron, can compute using theory and compare with experiment, get better than 1 part in 10^{12} agreement. A particular model, the "Standard Model", describes the strong, weak, electromagnetic interactions. What about gravity? Gravity is very different from other forces. Other forces carried by spin 1 object: photon, $W^{\pm}$, Z, gluons. Gravity is carried by a spin 2 object: the graviton $\sim \delta g_{\mu\nu}$. Spin 2 is very different than spin 1, especially at high energies!

Aside: the Higgs boson is the only known fundamental particle with spin 0. This is what makes it unusual, and also leads to conceptual challenges, e.g. naturalness vs fine tuning. Finding the Higgs and nothing else is something like finding a pencil balancing on its tip, with nothing to keep it balanced. This is what the LHC is telling us (so far).

• Quantum mechanics (or QFT) vs general relativity. Long standing clash. Write $G = 1/M_{pl}^2$ in $\hbar = c$ units. Quantum effects $\sim (GE^2)^\ell$, blow up for $E \sim M_{pl}$ ($E_{pl} = (\hbar c^5/G)^{1/2} = 1.22 \times 10^{19} GeV$) . Also many conceptual problems; black holes, meaning of quantum ideas when the metric itself can have quantum fluctuations.

String theory is the only known theory for resolving this clash, i.e. which gives a "UV completion" of quantum gravity. In string theory, replace point particles with tiny ($\ell \sim \ell_p = (G\hbar/c^3)^{1/2} = 1.62 \times 10^{-33} cm$) bits or loops of string. Turns out to lead to some bizarre consequences, like extra dimensions. Is it right? We don't know. At the very least, it is the only known well-defined theoretical framework which can be used to explore the mysteries of quantum gravity. Lessons learnt should be useful even if string theory isn't the last word on the subject. Has led to many interesting spin-offs and insights into topics which can be divorced from string theory, e.g. susy, new tools for strongly coupled gauge theories, holography (AdS/CFT).

• Curious history of string theory: originally developed to explain observed spectrum of mesons, e.g. $M^2 = (J + a)/\alpha'$.

But found that open strings always give massless spin 1 objects, and closed strings always give massless spin 2 objects. Mesons aren't like that. But massless spin 1 objects could be the photon and gluons – good! And massless spin 2 object could be the graviton – even better – Michael Green (Cambridge) and John Schwarz (Caltech) recycled the slightly off theory of mesons into a theory of quantum gravity! Mesons are described instead by QCD. (Still interest in QCD effective string theory.)

• Metric convention (mostly plus convention...sigh..) $x^\mu = (ct, x, y, z)$, $x_\mu = (-ct, x, y, z) = \eta_{\mu\nu} x^\mu$, $\eta_{\mu\nu}\eta^{\nu\lambda} = \delta_\mu^\lambda$. Define $ds^2 = -dx^\mu dx_\mu$. For 4-vectors $a^\mu = (a^0, \vec{a})$ then $a_\mu = (a_0 = -a^0, \vec{a})$. Define 4-vector dot products $a \cdot b \equiv a^\mu b^\nu \eta_{\mu\nu} = a^\mu b_\mu = a^\nu b_\nu = -a^0 b^0 + \vec{a} \cdot \vec{b}$. So $ds^2 \equiv -dx \cdot dx$ in this convention (sigh...).

• Δs^2 for time-like, light-like, space-like separated events. Statement of causality principle: cause's effects only in the time-like future.

• Statement of relativity principle: physics is indistinguishable among all inertial frames. If one frame is inertial, the other inertial frames have linearly related coordinates, $x^{\mu'} = \Lambda^{\mu'}{}_\nu x^\nu$, where the transformations must preserve $ds^2 = 0$; that is enough to show that they preserve any $ds^2 = ds'^2$; that is enough to show that they preserve all 4-scalar products. So $a \cdot b = a' \cdot b'$. This restricts the Lorentz transformations: if we write $\eta_{\mu\nu}$ as a matrix, the Lorentz transformations satisfy $\eta = \Lambda^T \eta \Lambda$. The Lorentz transformations consist of rotations and boosts (for a total of 3+3=6 independent generators). For the case of boosts, e.g. $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$, with $\gamma = (1 - v^2/c^2)^{-1/2}$ and $\beta = v/c$.

Aside on the Lorentz transformations (question from lecture): writing transformation in matrix notation, need to account for upper vs lower indices, e.g. $\eta^{\mu\nu}$ vs $\eta_{\mu\nu}$.

• $p^\mu = (E/c, p_x, p_y, p_z)$, with $p_\mu p^\mu = -m^2 c^2$. p^μ transforms as a Lorentz 4-vector, $p^{\mu'} = \Lambda^{\mu'}_\nu p^\nu$. Proper time: $ds^2 = c^2 dt_p^2 = c^2 dt^2 (1 - \beta^2)$. $u^\mu = c dx^\mu / ds = dx^\mu / dt_p = \gamma(c, \vec{v})$, and $u_\mu u^\mu = -c^2$. A massive point particle has $p^\mu = m u^\mu$. Massless particles, like the photon, have p^μ with $p^\mu p_\mu = 0$.

• Quantum mechanics: replace $p^\mu = (H/c, \vec{p}) \to -i\hbar \partial^\mu$. Free particle wavefunction $\psi \sim \exp(ip \cdot x / \hbar)$; $p_\mu x^\mu \equiv p \cdot x$ is Lorentz invariant.