6/4/19 Lecture outline

⋆ Reading: Zwiebach chapter 13

• Last time: finished with quantizing the open string (with Neumann BCs) in light cone gauge. The spacetime Lorentz symmetry required $D = 26$ and $M^2 = \frac{1}{\alpha'}\left(a + \sum_{n=1}^{\infty}\sum_{I=2}^{D-1} n a_n^{I\dagger} a_n^I\right)$ with $a = -1$. The states are $|\lambda\rangle = \prod_{n=1}^{\infty}\prod_{I=2}^{25}(a_n^{I\dagger})^{\lambda_{n,I}}|p^+, \vec{p}_T\rangle$.. The groundstate is tachyonic (the theory is unstable) and the first excited states are massless with $D-2$ polarizations, corresponding to photons. The tachyon is related to the fact that the D25 brane is unstable, it decays to the closed string vacuum. The closed bosonic string is also unstable, as we'll see today. These instabilities can be cured by adding fermions and considering the superstring. Then the critical spacetime dimension is $D = 10$, and $a = -1/2$.

• Now consider closed string case. Recall gauge conditions $n \cdot X = \alpha'(n \cdot p)\tau$, $n \cdot p = 2\pi n \cdot \mathcal{P}^\tau$, which yielded the constraints $(\dot{X} \pm X')^2 = 0$ and then the EOM were simply $(\partial_\tau^2 - \partial_\sigma^2)X^\mu = 0$. For the closed string, this means that $X^\mu(\tau,\sigma) = X_L^\mu(\tau+\sigma) + X_R^\mu(\tau-\sigma)$. The general solutions can then be written as

$$X_R^\mu(v) = \tfrac{1}{2}x_0^\mu + \sqrt{\tfrac{1}{2}\alpha'}\alpha_0^\mu v + i\sqrt{\tfrac{1}{2}\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-inv},$$

and a similar expression for X_L^μ, with modes $\widetilde{\alpha}_n^\mu$. Since $X^\mu(\tau,\sigma+2\pi) = X^\mu(\tau,\sigma)$, $\widetilde{\alpha}_0^\mu = \alpha_0^\mu$. Computing $\mathcal{P}^{\tau\mu} = \dot{X}^\mu/2\pi\alpha'$ then yields $\alpha_0^\mu = \sqrt{\frac{1}{2}\alpha'}p^\mu$.

As in the open string case, X^+ is proportional to τ and X^- can be solved for by the constraint, which gives $\dot{X}^- \pm X^{-'} = \frac{1}{2\alpha' p^+}(\dot{X}^I \pm X^{I'})^2$. The independent fields are X_L^I and X_R^I. The theory is quantized by taking $[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma')] = i\delta(\sigma-\sigma')\eta^{IJ}$, which implies that

$$[\alpha_m^I, \alpha_n^J] = m\delta_{m+n,0}\eta^{IJ}, \qquad [\widetilde{\alpha}_m^I, \widetilde{\alpha}_n^J] = m\delta_{m+n,0}\eta^{IJ}, \qquad [\alpha_m^I, \widetilde{\alpha}_n^J] = 0.$$

It's now very similar to the open string case, but with the two sets of decoupled oscillators for the left and right movers. We define

$$(\dot{X}^I + X'^I)^2 \equiv 4\alpha' \sum_n \widetilde{L}_n^\perp e^{-in(\tau+\sigma)},$$

and a similar expansion for $(\dot{X}^I - X'^I)^2$ and $L_n^\perp$, involving $\tau-\sigma$. Then $L_n^\perp = \frac{1}{2}\sum_p \alpha_p^I \alpha_{n-p}^I$, and $L_0^\perp = \frac{\alpha'}{4}p^I p^I + N^\perp$. As in the open string case, there is a normal ordering constant

$a = (D-2)/24$ that we separate off from $L_0^\perp$, and as in the open string case the Lorentz algebra requires $D = 26$ and $a = -1$. The X^- constraints give

$$\sqrt{2\alpha'}\alpha_n^- = \frac{2}{p^+} L_n, \qquad \sqrt{2\alpha'}\tilde{\alpha}_n^- = \frac{2}{p^+}\tilde{L}_n.$$

The worldsheet Hamiltonian is $H = L_0^\perp + \widetilde{L}_0^\perp - 2$ and the above relations for $n = 0$ give $M^2 = -p^2 = 2p^+p^- - p^I p^I = \frac{2}{\alpha'}(N^\perp + \widetilde{N}^\perp - 2)$.

The closed string states are given by acting with left and right moving creation operators on $|p^+, p^I\rangle$, with the constraint that $N^\perp = \widetilde{N}^\perp$ (because of translation symmetry in shifting σ). In summary, the spectrum of states is given by

$$\begin{gathered} |\lambda, \widetilde{\lambda}\rangle = [\prod_{n=1}^{\infty} \prod_{I=2}^{D-1} (a_n^{I\dagger})^{\lambda_{n,I}}][\prod_{n=1}^{\infty} \prod_{I=2}^{D-1} (\widetilde{a}_n^{I\dagger})^{\widetilde{\lambda}_{n,I}}]|p^\mu\rangle \\ M^2 = -p^2 = 2(N_\perp + \widetilde{N}_\perp - 2)/\alpha', \quad N^\perp = \sum_{n=1}^{\infty}\sum_{I=1}^{D-1} n\lambda_{n,I}, \quad N^\perp = \sum_{n=1}^{\infty}\sum_{I=1}^{D-1} n\widetilde{\lambda}_{n,I}, \end{gathered} \tag{1}$$

where there is a requirement that $N^\perp = \widetilde{N}^\perp$ to have σ translation invariance.

The state with $N^\perp = 0$ is the bosonic closed string tachyon. Those with $N^\perp = 1$ are given by a $(D-2)^2$ matrix of indices in the transverse directions, and these are massless. The symmetric traceless part is the graviton, the antisymmetric tensor is a gauge field $B_{\mu\nu}$ which is an analog of A_μ, and the trace part is ϕ, called the "dilaton." The D-dimensional Newton's constant is given by $G^{(D)} = \ell_P^{D-2} \sim g^2(\alpha')^{(D-2)/2}$. Get $g \sim e^\phi$.