137 HW 2 Due 4/16/19

1. Zwiebach 2.2

2. Equations of motion respect relativity if the action is relativistically invariant. Suppose that the action can be written as $S = \int dtL$ and $L = \int d^3x\mathcal{L}$, where $\mathcal{L}$ is the Lagrangian density. In this exercise, you will verify that S is a Lorentz scalar if $\mathcal{L}$ is a Lorentz scalar. To do this, you want to verify that $d^4x \equiv dx^0dx^1dx^2dx^3$ is invariant under Lorentz transformations. To do that, recall from your calculus class how, under changes of variables, the integration measure picks up a factor of the Jacobian determinant. Verify that the Jacobian determinant for $x^{\mu'} = \Lambda^{\mu'}_{\nu}x^\nu$ is equal to 1 for a boost along the x-axis and also for a rotation around the z axis. Show that $\Lambda^T\eta\Lambda = \eta$ implies that $\det\Lambda = \pm 1$, and that transformations that are continuously connected to the identity must have $\det\Lambda = 1$.

3. Using the fact that $j^\mu = (c\rho, \vec{J})$ transforms as a 4-vector, and the result of the previous question, argue that electric charge $Q = \int d^3x\rho$ is Lorentz invariant.

4. The action of a charged particle contains a term $S = \ldots + \frac{1}{c}\int d^4xA_\mu j^\mu$.

   (a) Taking $\rho = \sum_i q_i\delta^3(\vec{x} - \vec{x}_i(t))$ and $\vec{J} = \sum_i q_i\vec{v}_i(t)\delta^3(\vec{x} - \vec{x}_i(t))$ (where $\vec{v}_i = \dot{\vec{x}}_i$) verify that $\frac{1}{c}\int d^4xA_\mu j^\mu$ leads to the usual terms $L = \ldots + \sum_i q_i(\frac{1}{c}\vec{v}_i(t)\cdot\vec{A}_i - \phi(\vec{x}_i))$.

   (b) Explain why $\int d^4xA_\mu j^\mu$ is Lorentz invariant.

   (c) Verify that $\int d^4xA_\mu j^\mu$ is gauge invariant (dropping surface terms associated with integrals of total derivatives) provided that j^μ is a conserved current.