137 HW 1 Due 4/9/19

⋆ All numbered exercises are from Zwiebach

1. 2.3. In lecture, we discussed the Lorentz transformation Λ of a four vector a^μ under boosts with velocity v along the x-axis.

   (a) Verify that a_μ transforms by the inverse Λ^{-1}, which is related to Λ by $v \to -v$.

   (b) Using the chain rule, show that $\frac{\partial}{\partial x^\mu}$ transforms the same way as a_μ, with a lower index. So $\frac{\partial}{\partial x^\mu} \equiv \partial_\mu$.

   (c) Show that in quantum mechanics the expressions for the energy and momentum in terms of derivatives can be written as $p_\mu = \frac{\hbar}{i}\partial_\mu$.

   (d) QC 2.2: Let a^μ and b^μ be any 4-vectors. Transform a^μ and b^μ by a boost along the x-axis and verify that that $a^\mu b_\mu$ is invariant (this is similar to part (a)).

2. 2.4.

   (0) Verify that the Lorentz boost along the x-axis satisfies $\Lambda^T \eta \Lambda = \eta$, where η is the flat metric of spacetime. This is a general identity for Lorentz transformations: a transformation $a^{\mu'} = \Lambda^{\mu'}_{\nu} a^\nu$ is a Lorentz transformation if and only if the matrix $\Lambda^{\mu'}_{\nu}$ satisfies this identity.

   (a) Show that if Λ_1 and Λ_2 satisfy this identity, so does $\Lambda_1\Lambda_2$.

   (b) Show that if Λ satisfies this identity, so does Λ^{-1}.

   (c) Show that if Λ satisfies this identity, so does Λ^T.

3. Consider the spacetime path $x = x_0(\cosh\lambda - 1)$, $ct = x_0 \sinh\lambda$, where λ is a coordinate along the spacetime worldline of the object.

   (a) Compute the proper time $d\tau = c^{-1}\sqrt{-dx_\mu dx^\mu}$ for this path, and show that it is proportional to $d\lambda$, therefore λ is proportional to τ. Find the proportionality constant.

   (b) Compute $u^0 \equiv \frac{d(ct)}{d\tau}$ and $u^1 \equiv \frac{dx}{d\tau}$ and $v \equiv \frac{dx}{dt}$ for this path.