4/5/18 Lecture outline

⋆ Reading: Zwiebach chapters 1 and 2.

• Last time: Curious history of string theory: originally developed to explain observed spectrum of mesons, e.g. $M^2 = (J + a)/\alpha'$.

But found that open strings always give massless spin 1 objects, and closed strings always give massless spin 2 objects. Mesons aren't like that. But massless spin 1 objects could be the photon and gluons – good! And massless spin 2 object could be the graviton – even better – Michael Green (Cambridge) and John Schwarz (Caltech) recycled the slightly off theory of mesons into a theory of quantum gravity! Mesons are described instead by QCD. (Still interest in QCD effective string theory.)

• Metric convention (mostly plus convention...sigh..) $x^\mu = (ct, x, y, z)$, $x_\mu = (-ct, x, y, z) = \eta_{\mu\nu}x^\mu$, $\eta_{\mu\nu}\eta^{\nu\lambda} = \delta^\lambda_\mu$. Define $ds^2 = -dx^\mu dx_\mu$. For 4-vectors $a^\mu = (a^0, \vec{a})$ then $a_\mu = (a_0 = -a^0, \vec{a})$. Define 4-vector dot products $a \cdot b \equiv a^\mu b^\nu \eta_{\mu\nu} = a^\mu b_\mu = a^\nu b_\nu = -a^0 b^0 + \vec{a} \cdot \vec{b}$. So $ds^2 \equiv -dx \cdot dx$ in this convention (sigh...).

• Δs^2 for time-like, light-like, space-like separated events. Statement of causality principle: cause's effects only in the time-like future.

• Statement of relativity principle: physics is indistinguishable among all inertial frames. If one frame is inertial, the other inertial frames have linearly related coordinates, $x^{\mu'} = \Lambda^{\mu'}{}_\nu x^\nu$, where the transformations must preserve $ds^2 = 0$; that is enough to show that they preserve any $ds^2 = ds'^2$; that is enough to show that they preserve all 4-scalar products. So $a \cdot b = a' \cdot b'$. This restricts the Lorentz transformations: if we write $\eta_{\mu\nu}$ as a matrix, the Lorentz transformations satisfy $\eta = \Lambda^T \eta \Lambda$. The Lorentz transformations consist of rotations and boosts (for a total of 3+3=6 independent generators). For the case of boosts, e.g. $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$, with $\gamma = (1 - v^2/c^2)^{-1/2}$ and $\beta = v/c$.

Aside on the Lorentz transformations (question from lecture): writing transformation in matrix notation, need to account for upper vs lower indices, e.g. $\eta^{\mu\nu}$ vs $\eta_{\mu\nu}$.

• Light cone coordinates: $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The bad: spoils rotational symmetry. The good: will make it easier to quantize string theory (avoids having to discuss here more advanced alternatives, which do not require going to the light cone coordinates). $-ds^2 = -2dx^+ dx^- + dx_2^2 + dx_3^2 = -\widehat{\eta}_{\mu\nu} dx^\mu dx^\nu$. $a_\pm = -a^\mp$.

• $p^\mu = (E/c, p_x, p_y, p_z)$, with $p_\mu p^\mu = -m^2c^2$. p^μ transforms as a Lorentz 4-vector, $p^{\mu'} = \Lambda^{\mu'}_\nu p^\nu$. Proper time: $ds^2 = c^2 dt_p^2 = c^2 dt^2(1 - \beta^2)$. $u^\mu = c dx^\mu/ds = dx^\mu/dt_p = \gamma(c, \vec{v})$, and $u_\mu u^\mu = -c^2$. A massive point particle has $p^\mu = m u^\mu$. Massless particles, like the photon, have p^μ with $p^\mu p_\mu = 0$.

- Quantum mechanics: replace $p^\mu = (H/c, \vec{p}) \to -i\hbar\partial^\mu$. Free particle wavefunction $\psi \sim \exp(ip \cdot x/\hbar)$; $p_\mu x^\mu \equiv p \cdot x$ is Lorentz invariant.