5/29/18 Lecture outline

⋆ Reading: Zwiebach chapters 10 and 11.

• Last time: quantized the Klein-Gordon field, replacing ϕ with an operator. Consider

$$\phi(t,\vec{x}) = \frac{1}{\sqrt{V}}\sum_{\vec{p}} \frac{1}{\sqrt{2E_p}}(a_{\vec{p}}(t)e^{i\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}}(t)e^{-ip\cdot x}).$$

If we're in a spatial box, then $p_i L_i = 2\pi n_i$. Compute the energy to find

$$H = \sum_{\vec{p}>0}(\frac{1}{2E_p}\dot{a}^\dagger_p\dot{a}_p(t) + \tfrac{1}{2}E_p a^\dagger_p a_p) = \sum_{\vec{p}} E_p a^\dagger_{\vec{p}} a_{\vec{p}}.$$

where the EOM were used in the last step: $a_{\vec{p}}(t) = a_{\vec{p}}e^{-iE_pt} + a^\dagger_{-\vec{p}}e^{iE_pt}$. Also,

$$\vec{P} = \sum_{\vec{p}} \vec{p} a^\dagger_{\vec{p}} a_{\vec{p}}.$$

As expected, H and $\vec{P}$ are independent of t. We quantize this as a (complex) SHO for each value of $\vec{p}$:

$$[a_p, a_k^\dagger] = \delta_{p,k}, \qquad [a_p, a_k] = [a_p^\dagger, a_k^\dagger] = 0.$$

and interpret the above H and $\vec{P}$ has saying that $a^\dagger_{\pm\vec{p}}$ is a creation operator, creating a state with energy $E_p = \sqrt{\vec{p}^2+m^2}$ and spatial momentum $\vec{p}$ from the vacuum $|\Omega\rangle$. Note that we dropped the $2\cdot\frac{1}{2}E_p$ groundstate energy contribution, for no good reason. This is a contribution to the vacuum energy of empty space, and it is divergent upon summing over all p. This zero point energy is important (only) for gravity, and a contribution to the cosmological constant. Since this is an unresolved problem, we won't discuss it further.

• Now consider the Maxwell field A^μ and quantize → photons. In the vacuum, setting $j^\mu = 0$, we have $\partial_\mu F^{\nu\mu} = 0$, which implies $\partial^2 A^\mu - \partial^\mu(\partial\cdot A) = 0$. Massless. Fourier transform to $A^\mu(p)$, with $A^\mu(-p) = A^\mu(p)^*$, and get $(p^2\eta^{\mu\nu} - p^\mu p^\nu)A_\nu(p) = 0$. Gauge invariance $\delta A_\mu(p) = ip_\mu\epsilon(p)$. In light cone gauge, since $p^+\neq 0$, can set $A^+(p) = 0$. Then get $A^- = (p^I A^I)/p^+$, i.e. A^- is not an independent d.o.f., but rather constrained, and the Maxewell EOM gives $p^2 A^\mu(p) = 0$. For $p^2\neq 0$, require $A^\mu(p) = 0$, and for $p^2 = 0$ get that there are $D-2$ physical transverse d.o.f., the $A^I(p)$. The one-photon states are

$$\sum_{I=2}^{D-1} \xi_I a^{I\dagger}_{p^+,p_T}|\omega\rangle.$$

Gravitational light cone gauge conditions: $h^{++} = h^{+-} = h^{+I} = 0$. Other light cone components are constrained. The equations of motion, with $p_+ \neq 0$, imply that $h^{ij}\delta_{IJ} = 0$. So physical d.o.f. are specified by a traceless symmetric matrix h^{IJ} in the $D-2$ transverse directions. So $\frac{1}{2}D(D-3)$ d.o.f..

• Recall the relativistic point particle, with $S = \int L d\tau$ and $L = -m\sqrt{-\dot{x}^2}$, where $\dot{\equiv} \frac{d}{d\tau}$. ($\tau$ is taken to be dimensionless.) The momentum is $p_\mu = \partial L/\partial \dot{x}^\mu = m\dot{x}_\mu/\sqrt{-\dot{x}^2}$ and the EOM is $\dot{p}_\mu = 0$. In light cone gauge we take $x^+ = p^+\tau/m^2$. Then $p^+ = m\dot{x}^+/\sqrt{-\dot{x}^2}$ and the light cone gauge condition implies $\dot{x}^2 = -1/m^2$, so $p_\mu = m^2\dot{x}_\mu$. Also, $p^2 + m^2 = 0$ yields $p^- = (p^I p^I + m^2)/2p^+$, which is solved for p^- and then $\dot{x}^- = p^-/m^2$ is integrated to $x^- = p^-\tau/m^2 + x_0^-$. Also, $x^I = x_0^I + p^I\tau/m^2$. The dynamical variables are (x^I, x_0^-, p^I, p^+).